The Use of Signal Filtering for Hog Inventory Estimation

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National Agricultural Statistics Service

“... providing timely, accurate, and useful statistics in service to U.S. agriculture.”
Agency Overview

• The USDA's National Agricultural Statistics Service (NASS) conducts hundreds of surveys every year and prepares reports covering virtually every aspect of U.S. agriculture. Production and supplies of food and fiber, prices paid and received by farmers, farm labor and wages, farm finances, chemical use, and changes in the demographics of U.S. producers are only a few examples.
Presentation Goals

• Convey Complexity of Hog Inventory Estimation Problem

• Present Solution of Inventory Estimation Using Signal Filtering Techniques

• Address How Signal Filtering Applies to the Topic of Weighting

• Share Some Supporting Results
Hog Inventory Estimation Problem

• 10 Inventory Types at U.S. and State levels

• Inventory Transaction Data (External-U.S. only)

• Multiple Survey Measurements (Internal)

• Set of Assumed Constraints
Hog Inventory Types

- Total Hogs & Pigs
- Pig Crop (Births)
- Sows Farrowed
- Market Hogs
- Breeding Herd
- Litter Rate
Hog Inventory Transaction Data

- Imports
- Exports
- Canadian Feeder Pigs
- Farm Slaughter
- Commercial Slaughter
- Death Loss
Hog Survey Measurements

- List Frame (ADXX)
- Multiframe (ADMW)
- Reweighted (RWMW)
## Hog Inventory Constraints

<table>
<thead>
<tr>
<th>Inventory Related Item</th>
<th>Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>total (H)ogs and pigs</td>
<td>H</td>
</tr>
<tr>
<td>(P)ig crop</td>
<td>P</td>
</tr>
<tr>
<td>(S)ows farrowed</td>
<td>S</td>
</tr>
<tr>
<td>s(L)aughter</td>
<td>L</td>
</tr>
<tr>
<td>(M)arket hogs</td>
<td>M</td>
</tr>
<tr>
<td>(I)mports-(E)xports-(D)eaths-L</td>
<td>BSN (Balance Sheet Net)</td>
</tr>
</tbody>
</table>

\[
\hat{X}_t := \text{Survey estimate for } X \text{ at time } t.
\]
Hog Inventory Constraints

• Survey estimates are biased.

\[
\begin{align*}
E\hat{H} & \neq H \\
E\hat{P} & \neq P \\
E\hat{S} & \neq S
\end{align*}
\]
Hog Inventory Constraints

• Survey litter rate is an unbiased approximation of true litter rate.

\[ E \left( \frac{\hat{P}}{\hat{S}} \right) \approx \frac{P}{S} \]
Hog Inventory Constraints

• Annual increase in s(L)aughter is unbiased approximation of annual increase in (P)ig crop two quarters in the past.

\[
E \left[ \frac{\hat{L}_t}{\hat{L}_{t-4}} \right] \approx \frac{P_{t-2}}{P_{t-6}}
\]
Hog Inventory Constraints

• Annual increase in 6 months s(L)laughter is unbiased approximation of annual increase in total (M)arket hogs two quarters in the past.

\[
E \left[ \frac{\hat{L}_t + \hat{L}_{t-1}}{\hat{L}_{t-4} + \hat{L}_{t-5}} \right] \approx \frac{M_{t-2}}{M_{t-6}}
\]
Hog Inventory Constraints

- Balance Sheet Residual

\[ H_t = H_{t-1} + P_t + BSN_t \]
Hog Inventory Constraints

• Balance Sheet Residual

\[ H_t = H_{t-1} + P_t + BSN_t \]

\[ |H_t - H_{t-1} - P_t - BSN| \leq 500,000 \]
Agricultural Statistics Board (ASB) Measurements

- Inventory Transaction Data
- Survey Measurements
- Assumed Constraints

Agricultural Statistics Board (ASB) Estimates
ASB Inventory Estimation Process

Survey Measurements

Inventory Transaction Data

Assumed Constraints

ASB Estimates
Solution Through Signal Filtering

- Inventory Transaction Data
- Assumed Constraints
- Survey Measurements
- ASB Measurements

Signal Filtering (Kalman Filter) Estimates
Signal Filtering: Definition

• Process of measuring signals obscured by noise.

Applications: Target Tracking

• Sonar
• Radar
• GPS
Signal Filtering: Example
Signal Filtering: Example
Signal Filtering: Example

- Trend
- Periodic Component
- White Noise Process
Kalman Filter Concepts

• Let $\mathbf{Y}$ be a vector of observations or measurements (Survey, ASB, Transaction Data).

• Let $\mathbf{X}$ be a vector representing an unobserved signal (Hog Inventories).
State Space System Equations

\[ X_t = \Phi X_{t-1} + W_t \]

• Transition Equation
  – Transition Model
  – Transition Assumption Constraints

\[ Y_t = A_t X_t + V_t \]

• Measurement Equation
  – Survey, ASB measurements
  – External Transaction Data
State Space System Equations

• Prediction

\[ E \begin{bmatrix} K_t \end{bmatrix} \bar{Y}_{t-1} \]

• Update

\[ E \begin{bmatrix} K_t \end{bmatrix} \bar{Y}_t \]
Kalman Filter Equations

• Prediction

\[ X_{t|t-1} = E \left[ K_t \mid Y_{t-1} \right] = \Phi X_{t-1|t-1} \]

• Update

\[ X_{t|t} = E \left[ K_t \mid Y_t \right] = X_{t|t-1} + K_t \left[ t - A_t X_{t|t-1} \right] \]
Kalman Filter Equations

• Prediction

\[ X_{t|t-1} = E \left[ K_t \mid Y_{t-1} \right] = \Phi X_{t-1|t-1} \]

• Update

\[ X_{t|t} = E \left[ K_t \mid Y_t \right] = X_{t|t-1} + K_t \left( y_t - A_t X_{t|t-1} \right) \]

Multivariate Normal Conditional Distribution

\[ E \left[ z_{2|1} \right] = a \mid z_{2} = \mu_1 + \sum_{12} \sum_{22}^{-1} \left( a - \mu_2 \right) \]
Kalman Filter Equations

\[ X_{t|t} = X_{t|t-1} + K_t \left( z_t - A_t X_{t|t-1} \right) \]
Kalman Filter Equations

\[ X_{t|t} = X_{t|t-1} + K_t \left( z_t - A_t X_{t|t-1} \right) \]
\[ = -K_t A_t \tilde{X}_{t|t-1} + K_t Y_t \]
Kalman Filter Equations

\[
X_{t|t} = X_{t|t-1} + K_t \left( \hat{x}_{t} - A_t X_{t|t-1} \right) \\
= \left( -K_t A_t \right) \hat{X}_{t|t-1} + K_t Y_t \\
= \left( -K_t A_t \right) \Phi X_{t-1|t-1} + K_t Y_t
\]
Kalman Filter Equations

\[ X_{t|t} = X_{t|t-1} + K_t \left[ Y_t - A_t X_{t|t-1} \right] \]

\[ = \left[ -K_t A_t \right] X_{t|t-1} + K_t Y_t \]

\[ = \left[ -K_t A_t \right] \Phi X_{t-1|t-1} + K_t Y_t \]

\[ = \left[ -K_t A_t \right] \Phi \left[ -K_{t-1} A_{t-1} \right] \Phi X_{t-2|t-2} + \left[ -K_t A_t \right] \Phi K_{t-1} Y_{t-1} + K_t Y_t \]
Kalman Filter Equations

\[ X_{t|t} = X_{t|t-1} + K_t (\hat{X}_t - A_t X_{t|t-1}) \]

\[ = (I - K_t A_t) X_{t|t-1} + K_t Y_t \]

\[ = (I - K_t A_t) \Phi X_{t-1|t-1} + K_t Y_t \]

\[ = (I - K_t A_t) \Phi (I - K_{t-1} A_{t-1} \Phi) X_{t-2|t-2} + (I - K_t A_t) \Phi K_{t-1} Y_{t-1} + K_t Y_t \]

\[ = \left[ \prod_{k=0}^{t-1} (I - K_{t-k} A_{t-k} \Phi) \right] x_{0|0} + \sum_{m=0}^{t-1} \left[ \prod_{j=1}^{m} (I - K_{t-j+1} A_{t-j+1} \Phi) \right] K_{t-m} Y_{t-m} \]
Kalman Filter Equations

\[ x_{t|t} = \sum_{m=0}^{t} \Psi_m Y_{t-m} \mid Y_0 = x_{0|0} \]

\[ \Psi_m = \begin{cases} 
K_t & m = 0 \\
\prod_{j=1}^{m} \left( -K_{t-j+1} A_{t-j+1} \phi \right) K_{t-m} & 0 < m < t \\
\prod_{k=0}^{t-1} \left( -K_{t-k} A_{t-k} \phi \right) & m = t 
\end{cases} \]
Kalman Filter Equations

\[ x_{t|t} = K_t Y_t + \sum_{m=1}^{t} \Psi_m Y_{t-m} \]

- Signal at time “t”
- Measurement Contribution at time “t”
- Cumulative Past Contributions
Filter Results Comparing ASB Parameterization

- Exclude ASB
- Include
  - Biased
  - Unbiased

\[
E \left[ \hat{X}^{ASB}_t \right] = X_t + b_t \\
E \left[ \hat{X}^{ASB}_t \right] = X_t
\]
Filter Results: Total Hogs & Pigs

ASB Biased

ASB Unbiased
Filter Results: Total Hogs & Pigs

Contribution to Last Estimate: ASB Biased

Contribution to Last Estimate: ASB Unbiased
Filter Results

ASB Biased

12 Month Balance Sheet Residual

Year
-4000 -2000 0 2000 4000
Published
Kalman Filter
Bounds: (-500,500)

ASB Unbiased

12 Month Balance Sheet Residual

Year
-4000 -2000 0 2000 4000
Published
Kalman Filter
Bounds: (-500,500)
Additional Topics

• Handling of ASB Measurements
  – Include
    • Biased
    • Unbiased
  – Exclude

• Restricted Least Squares State Allocation

• Comparison of Kalman Filter estimates under various parameterizations
References

