The Use of Signal Filtering for Hog Inventory Estimation

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National Agricultural Statistics Service

Agency Overview

- The USDA's National Agricultural Statistics Service (NASS) conducts hundreds of surveys every year and prepares reports covering virtually every aspect of U.S. agriculture. Production and supplies of food and fiber, prices paid and received by farmers, farm labor and wages, farm finances, chemical use, and changes in the demographics of U.S. producers are only a few examples.

Presentation Goals

- Convey Complexity of Hog Inventory Estimation Problem
- Present Solution of Inventory Estimation Using Signal Filtering Techniques
- Address How Signal Filtering Applies to the Topic of Weighting
- Share Some Supporting Results

Hog Inventory Estimation Problem

- 10 Inventory Types at U.S. and State levels
- Inventory Transaction Data (External-U.S. only)
- Multiple Survey Measurements (Internal)
- Set of Assumed Constraints

Hog Inventory Types

- Total Hogs & Pigs
- Pig Crop (Births)
- Sows Farrowed
- Market Hogs
- Breeding Herd
- Litter Rate

Hog Inventory Transaction Data

- Imports
- Exports
- Canadian Feeder Pigs
- Farm Slaughter
- Commercial Slaughter
- Death Loss

Hog Survey Measurements

- List Frame (ADXX)
- Multiframe (ADMW)
- Reweighted (RWMW)

Hog Inventory Constraints

Inventory Related Item	Notation
total (H)ogs and pigs	H
(P)ig crop	P
(S)ows farrowed	S
s(L)aughter	L
(M)arket hogs	M
(I)mports-(E)xports-(D)eaths-L	BSN (Balance Sheet Net)

$$\hat{X}_t := \text{Survey estimate for X at time } t.$$

Hog Inventory Constraints

- Survey estimates are biased.

$$E\left[\hat{H}\right] \neq H$$

$$E\left[\hat{P}\right] \neq P$$

$$E\left[\hat{S}\right] \neq S$$

Hog Inventory Constraints

- Survey litter rate is an unbiased approximation of true litter rate.

$$E\left[\frac{\hat{P}}{\hat{S}}\right] \approx \frac{P}{S}$$

Hog Inventory Constraints

- Annual increase in s(L)aughter is unbiased approximation of annual increase in (P)ig crop two quarters in the past.

$$E\left[\frac{\hat{L}_t}{\hat{L}_{t-4}}\right] \approx \frac{P_{t-2}}{P_{t-6}}$$

Hog Inventory Constraints

- Annual increase in 6 months s(L)aughter is unbiased approximation of annual increase in total (M)arket hogs two quarters in the past.

$$E\left[\frac{\hat{L}_t + \hat{L}_{t-1}}{\hat{L}_{t-4} + \hat{L}_{t-5}}\right] \approx \frac{M_{t-2}}{M_{t-6}}$$

Hog Inventory Constraints

- Balance Sheet Residual

$$H_t = H_{t-1} + P_t + BSN_t$$

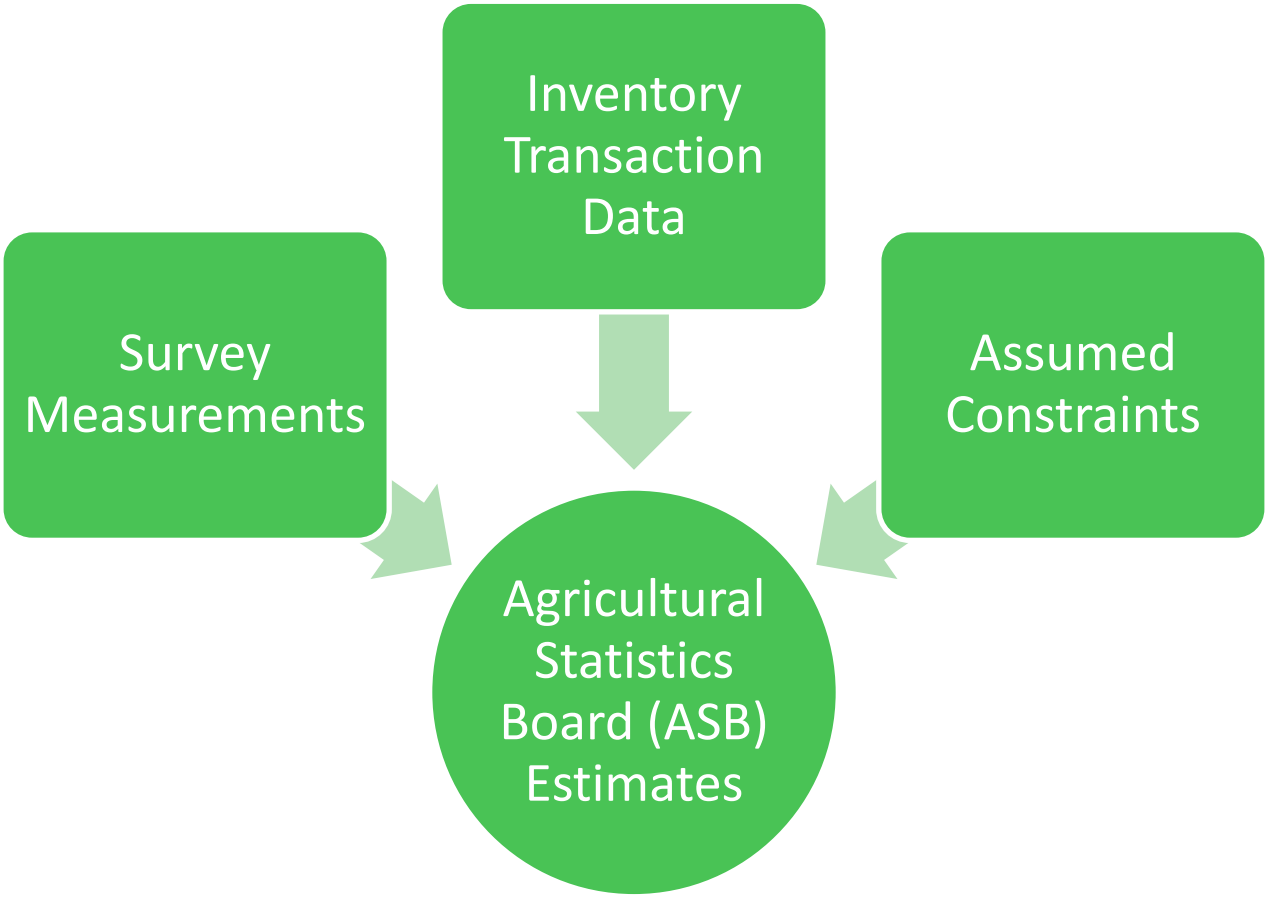
Hog Inventory Constraints

- Balance Sheet Residual

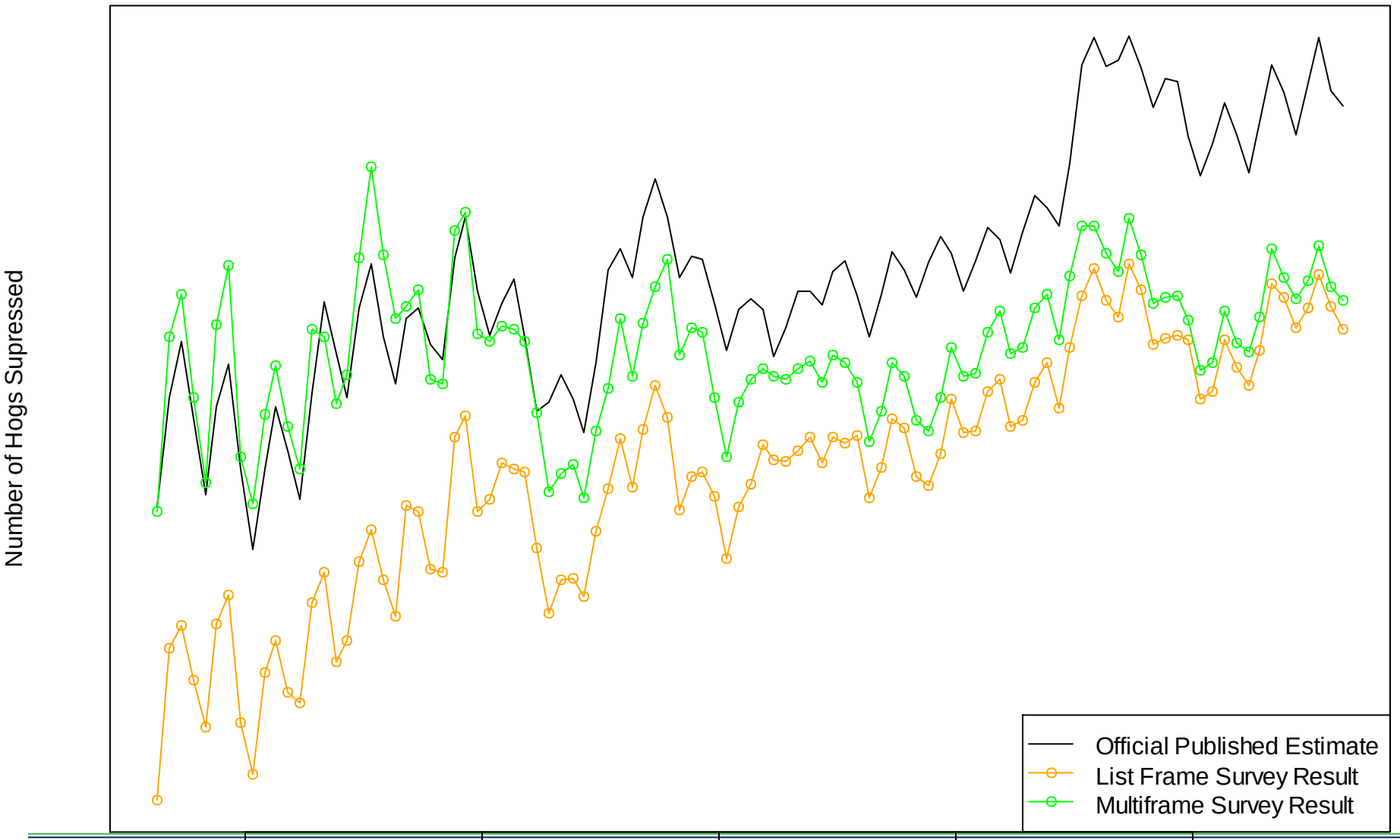
$$H_t = H_{t-1} + P_t + BSN_t$$

$$\left| H_t - H_{t-1} - P_t - BSN \right| \leq 500{,}000$$

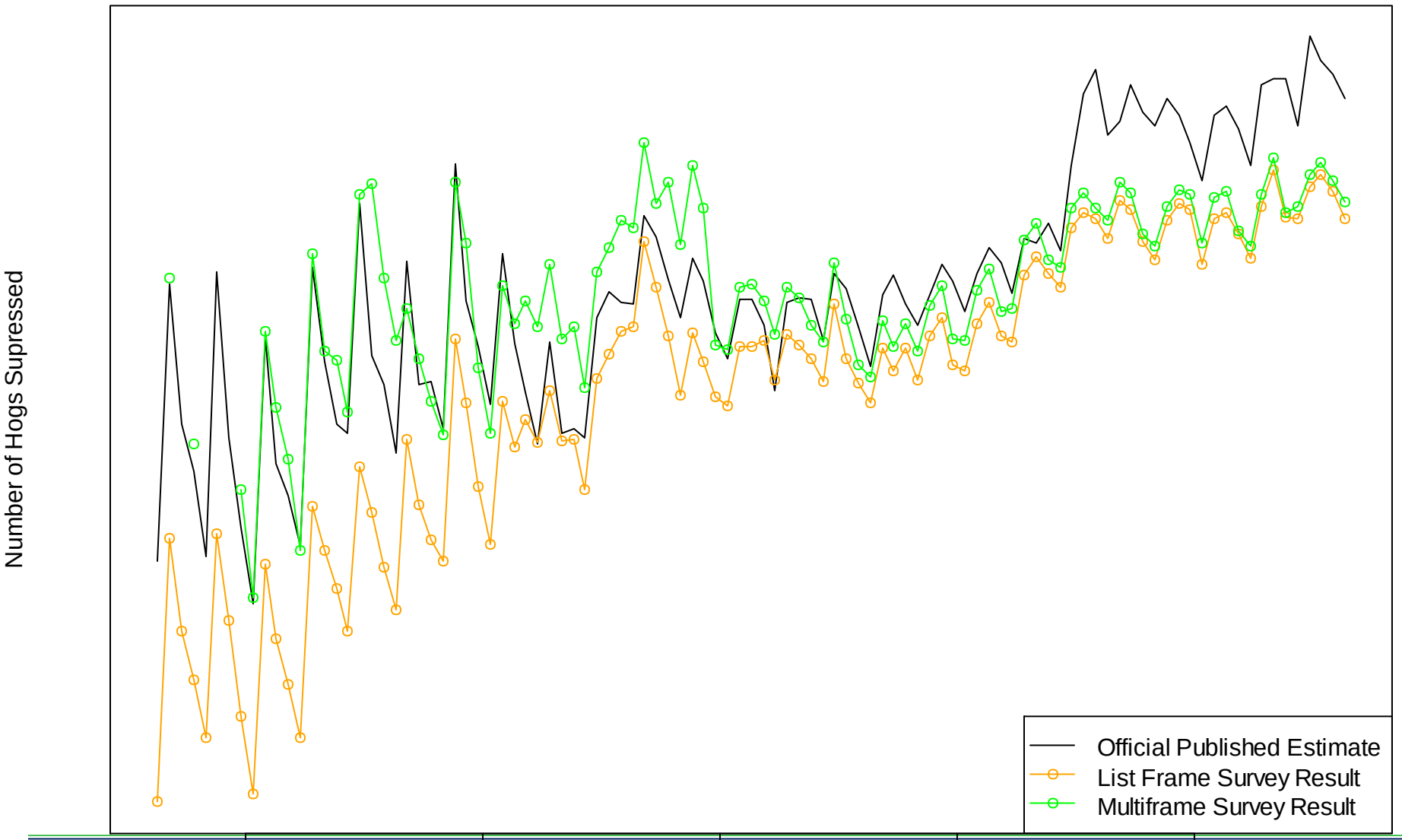
Agricultural Statistics Board (ASB) Measurements



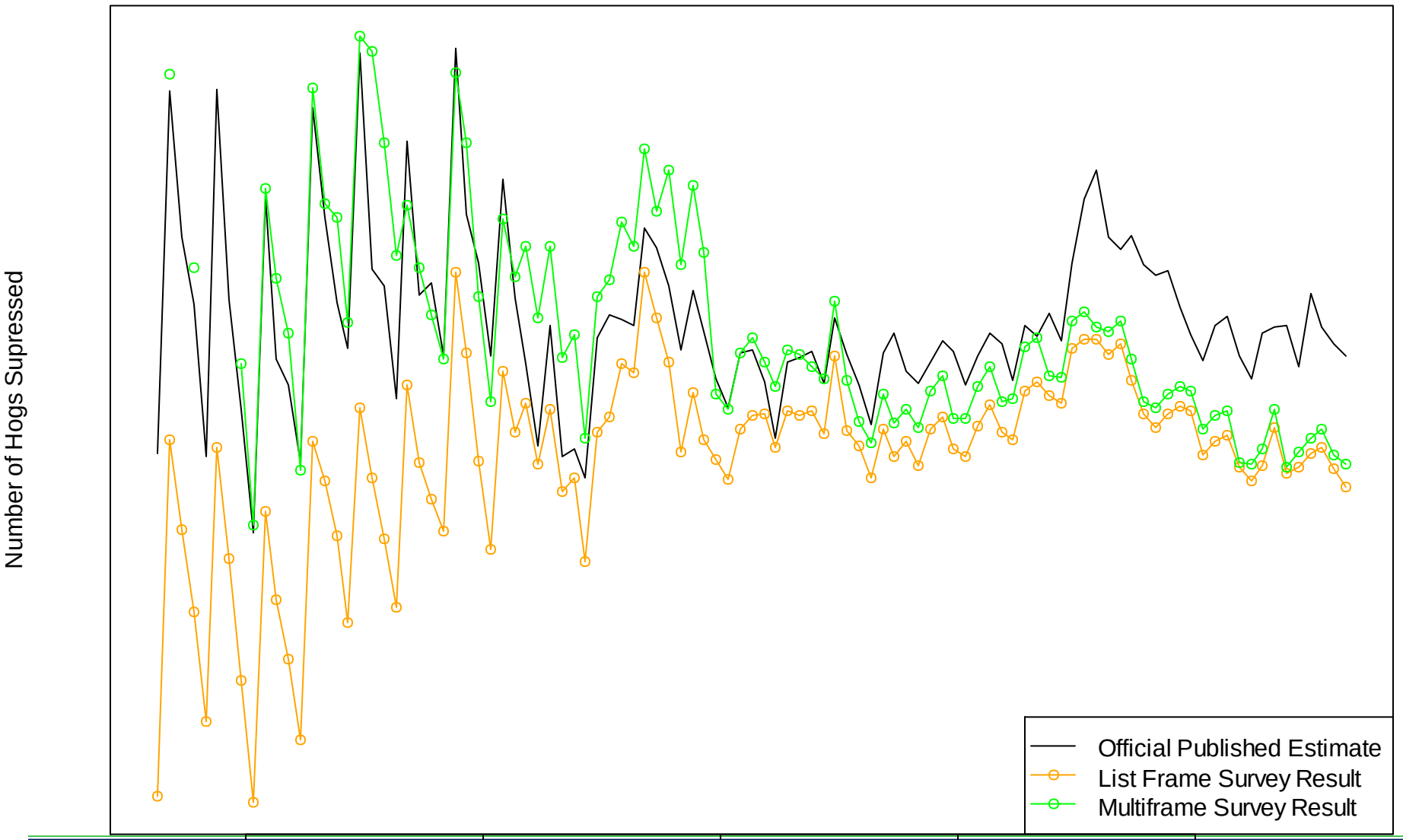
Total Hogs Inventory



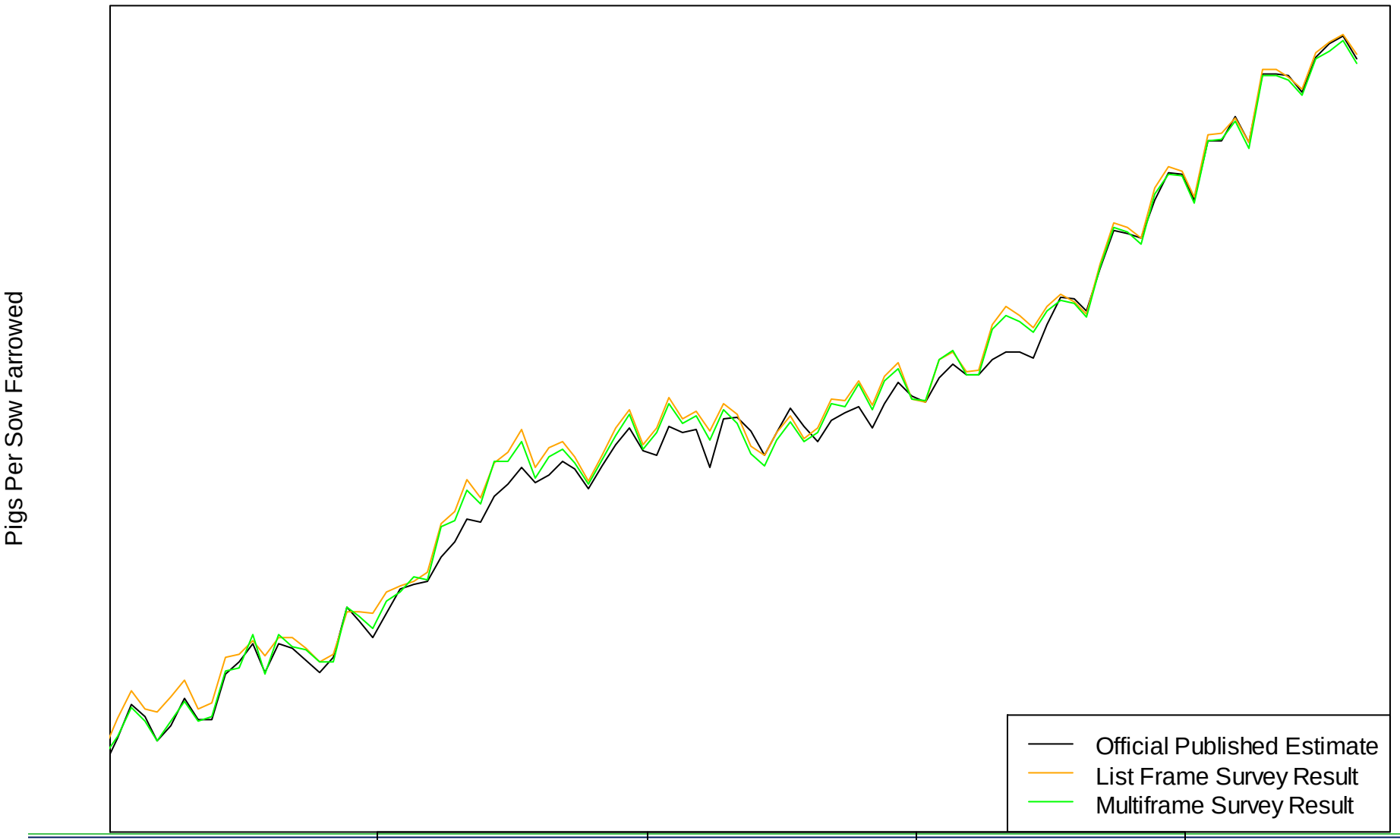
Pig Crop (Births) Inventory



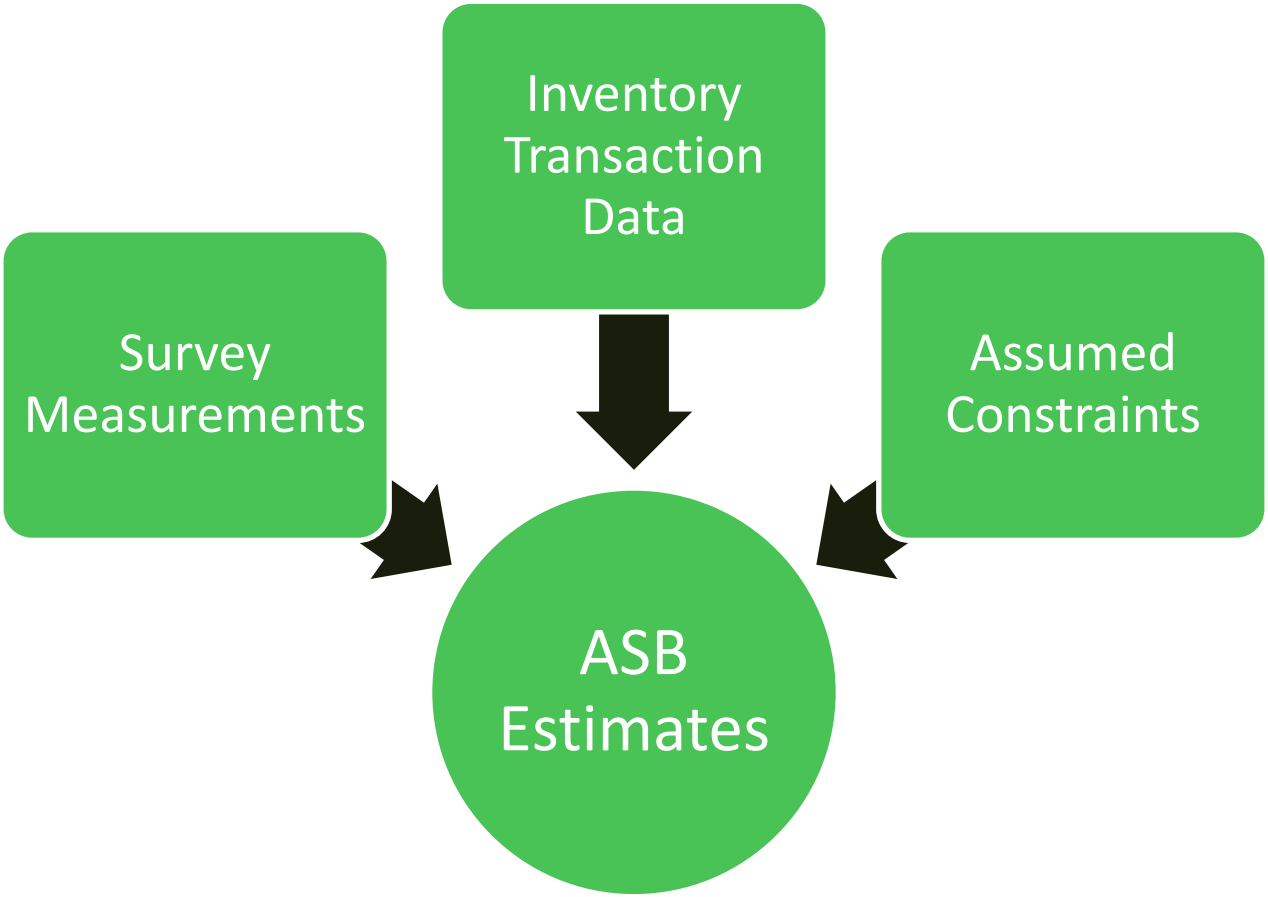
Sows Farrowed Inventory



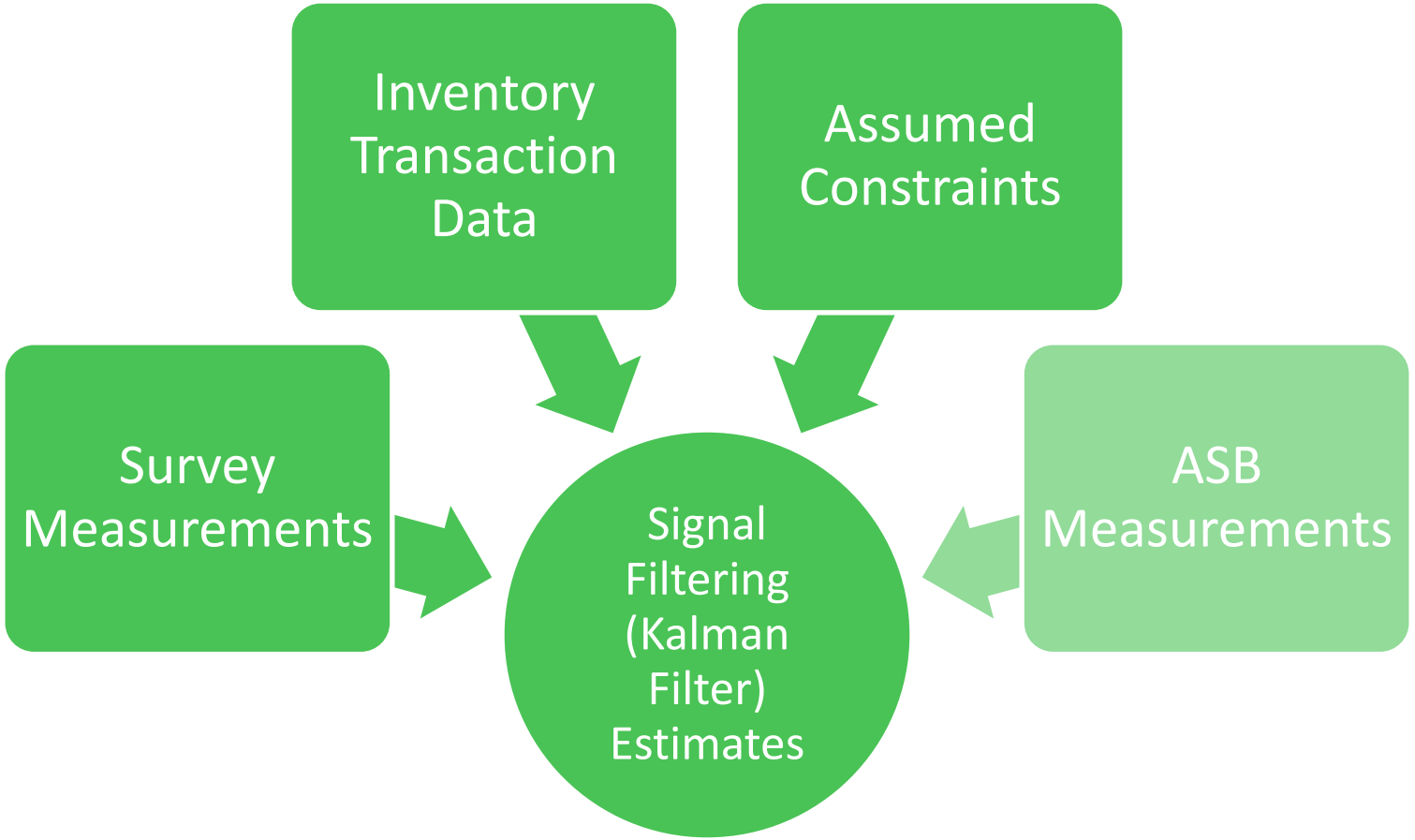
Litter Rate


Pigs Per Sow Farrowed
1995
2000
2005
2010
Official Published Estimate
List Frame Survey Result
Multiframe Survey Result

ASB Inventory Estimation Process



Solution Through Signal Filtering



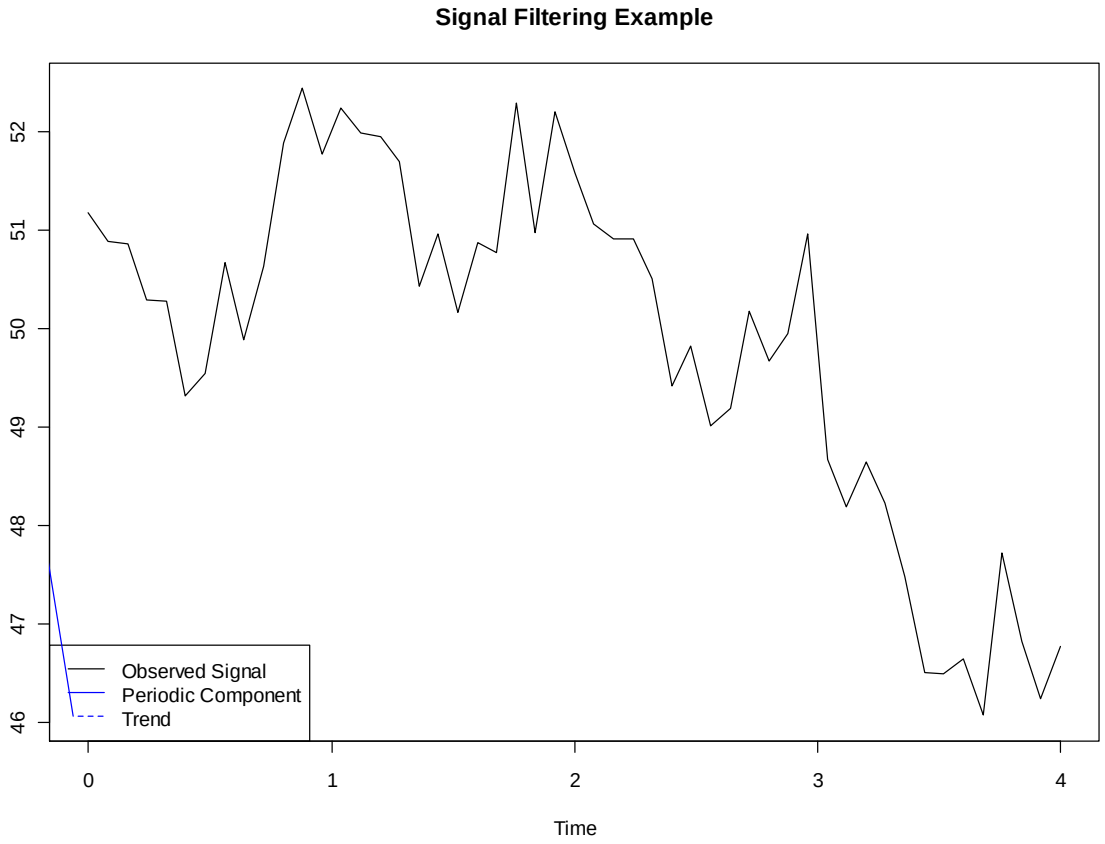
Signal Filtering: Definition

- Process of measuring signals obscured by noise.

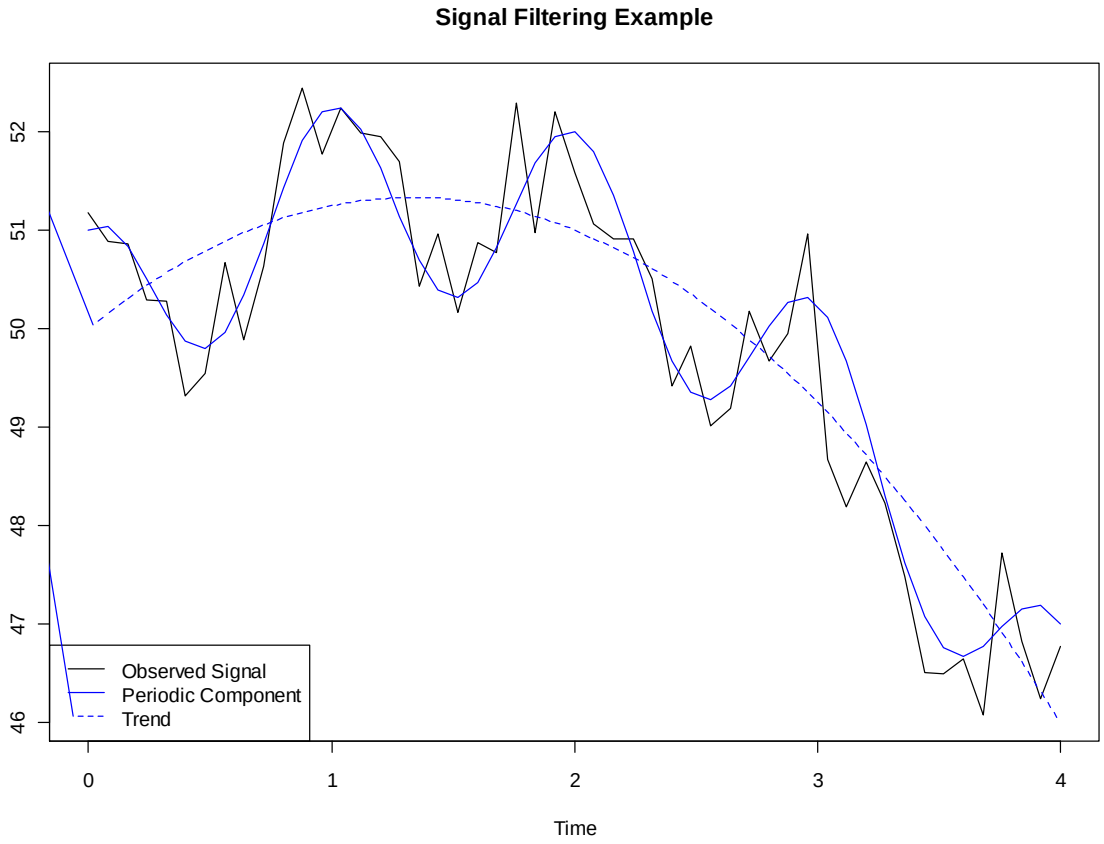
<u>Applications: Target Tracking</u>

- Sonar
- Radar
- GPS

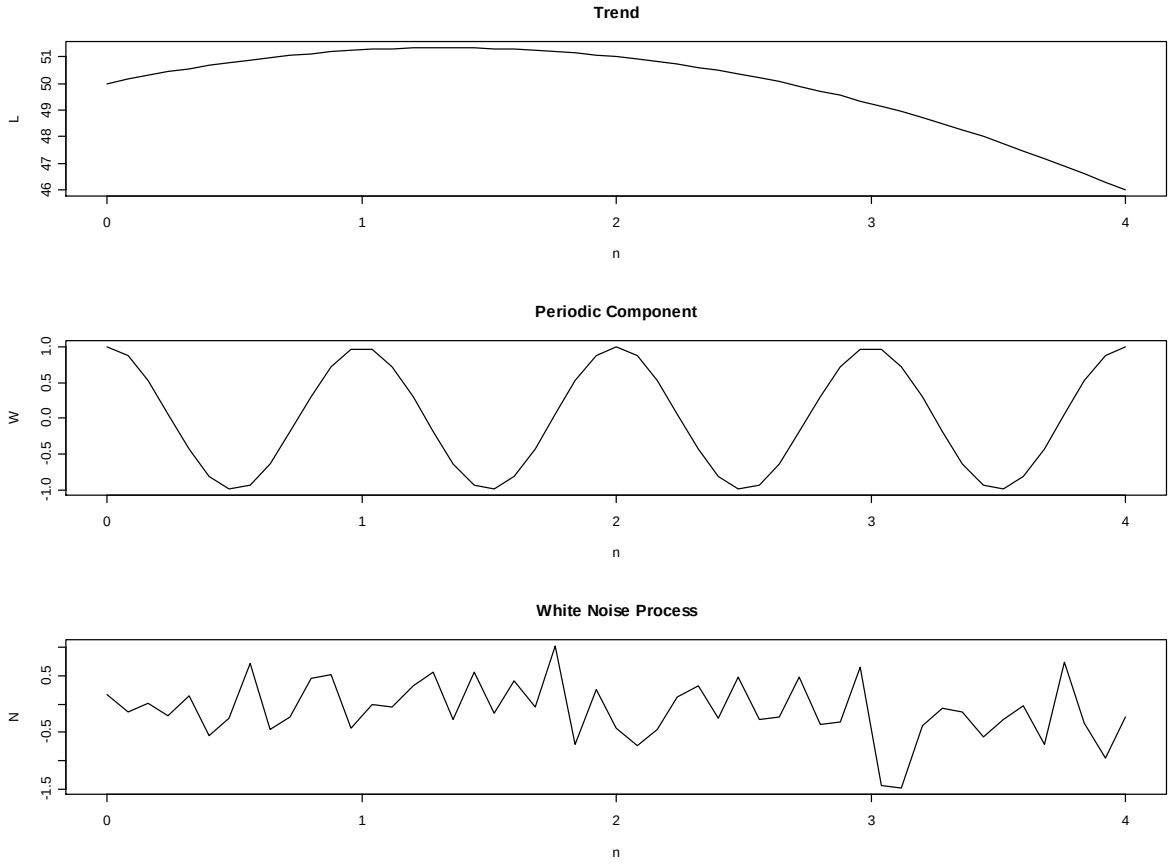
Signal Filtering: Example



Signal Filtering: Example



Signal Filtering: Example



Kalman Filter Concepts

- Let **Y** be a vector of observations or measurements (Survey, ASB, Transaction Data).

- Let **X** be a vector representing an unobserved signal (Hog Inventories).

State Space System Equations

$$X_t = \Phi X_{t-1} + W_t$$

- Transition Equation
  - Transition Model
  - Transition Assumption Constraints

$$Y_t = A_t X_t + V_t$$

- Measurement Equation
  - Survey, ASB measurements
  - External Transaction Data

State Space System Equations

- Prediction

$$E\left[X_t \mid Y_{t-1}\right]$$

- Update

$$E\left[X_t \mid Y_t\right]$$

Kalman Filter Equations

- Prediction

$$X_{t|t-1} = E\left[X_t \mid Y_{t-1}\right] = \Phi X_{t-1|t-1}$$

- Update

$$X_{t|t} = E\left[X_t \mid Y_t\right] = X_{t|t-1} + K_t\left(Y_t - A_t X_{t|t-1}\right)$$

Kalman Filter Equations

- Prediction

$$X_{t|t-1} = E\left[X_t \mid Y_{t-1}\right] = \Phi X_{t-1|t-1}$$

- Update

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Multivariate Normal Conditional Distribution

$$E\left[z_1 \mid z_2 = a\right] = \mu_1 + \Sigma_{12}\Sigma_{22}^{-1}\left(a - \mu_2\right)$$

Kalman Filter Equations

$$X_{t|t} = X_{t|t-1} + K_t \left(Y_t - A_t X_{t|t-1} \right)$$

Kalman Filter Equations

$$X_{t|t} = X_{t|t-1} + K_t \left(Y_t - A_t X_{t|t-1} \right)$$
$$= \left(I - K_t A_t \right) X_{t|t-1} + K_t Y_t$$

Kalman Filter Equations

$$
\begin{aligned}
X_{t|t} &= X_{t|t-1} + K_t\left(Y_t - A_t X_{t|t-1}\right) \\
&= \left(I - K_t A_t\right) X_{t|t-1} + K_t Y_t \\
&= \left(I - K_t A_t\right) \Phi X_{t-1|t-1} + K_t Y_t
\end{aligned}
$$

Kalman Filter Equations

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\begin{aligned}
X_{t|t} &= X_{t|t-1} + K_t\left(Y_t - A_t X_{t|t-1}\right) \\
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&= \left(I - K_t A_t\right) \Phi X_{t-1|t-1} + K_t Y_t \\
&= \left(I - K_t A_t\right) \Phi \left(I - K_{t-1} A_{t-1}\right) \Phi X_{t-2|t-2} + \left(I - K_t A_t\right) \Phi K_{t-1} Y_{t-1} + K_t Y_t
\end{aligned}
$$

Kalman Filter Equations

$$
\begin{aligned}
X_{t|t} &= X_{t|t-1} + K_t\left(Y_t - A_t X_{t|t-1}\right) \\
&= \left(I - K_t A_t\right) X_{t|t-1} + K_t Y_t \\
&= \left(I - K_t A_t\right) \Phi X_{t-1|t-1} + K_t Y_t \\
&= \left(I - K_t A_t\right) \Phi \left(I - K_{t-1} A_{t-1}\right) \Phi X_{t-2|t-2} + \left(I - K_t A_t\right) \Phi K_{t-1} Y_{t-1} + K_t Y_t \\
&= \left[\prod_{k=0}^{t-1} \left(I - K_{t-k} A_{t-k}\right) \Phi\right] x_{0|0} + \sum_{m=0}^{t-1} \left[\prod_{j=1}^{m} \left(I - K_{t-j+1} A_{t-j+1}\right) \Phi\right] K_{t-m} Y_{t-m}
\end{aligned}
$$

Kalman Filter Equations

$$x_{t|t} = \sum_{m=0}^{t} \Psi_m Y_{t-m} \mid Y_0 = x_{0|0}$$

$$\Psi_m = \begin{cases} K_t & m = 0 \\ \left[\prod_{j=1}^{m} \left(- K_{t-j+1} A_{t-j+1} \right) \Phi \right] K_{t-m} & 0 < m < t \\ \prod_{k=0}^{t-1} \left(- K_{t-k} A_{t-k} \right) \Phi & m = t \end{cases}$$

Kalman Filter Equations

$$x_{t|t} = K_t Y_t + \sum_{m=1}^{t} \Psi_m Y_{t-m}$$

Signal at time "t"

Measurement Contribution at time "t"

Cumulative Past Contributions

Filter Results Comparing ASB Parameterization

- Exclude ASB
- Include
  - Biased

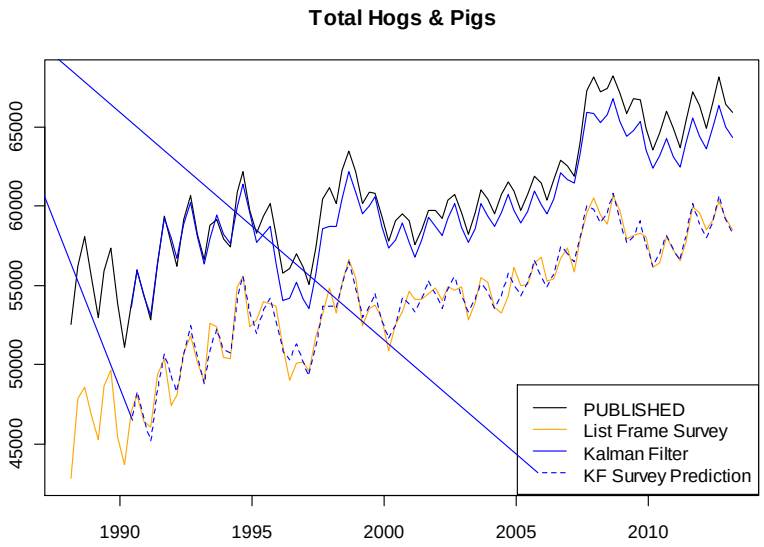
$$E\left[\hat{X}_t^{ASB}\right] = X_t + b_t$$

  - Unbiased

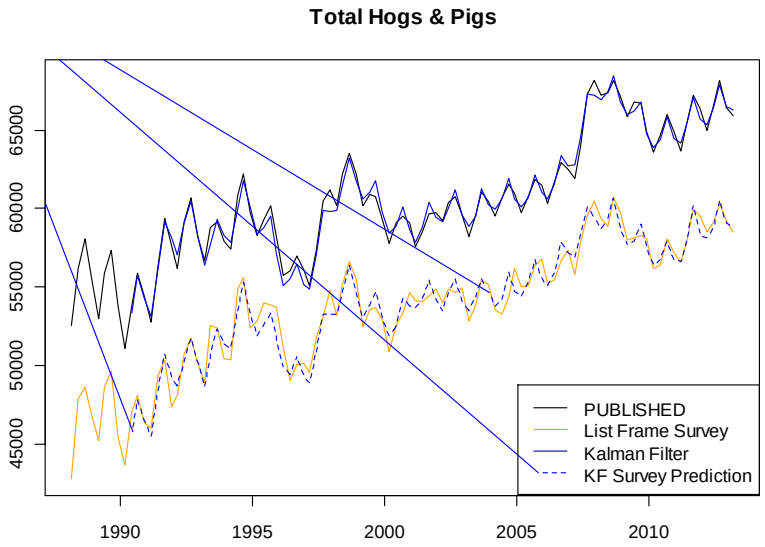
$$E\left[\hat{X}_t^{ASB}\right] = X_t$$

Filter Results: Total Hogs & Pigs

ASB Biased

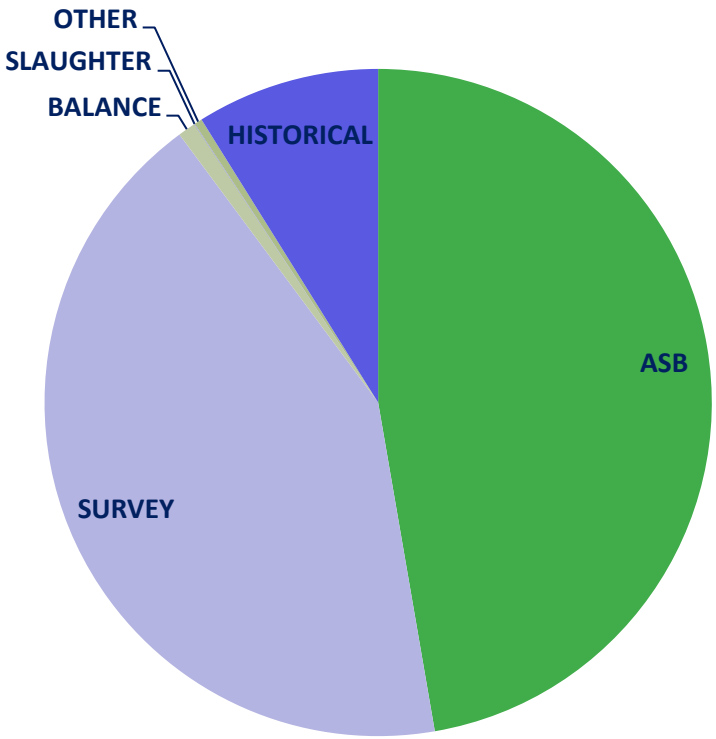


ASB Unbiased

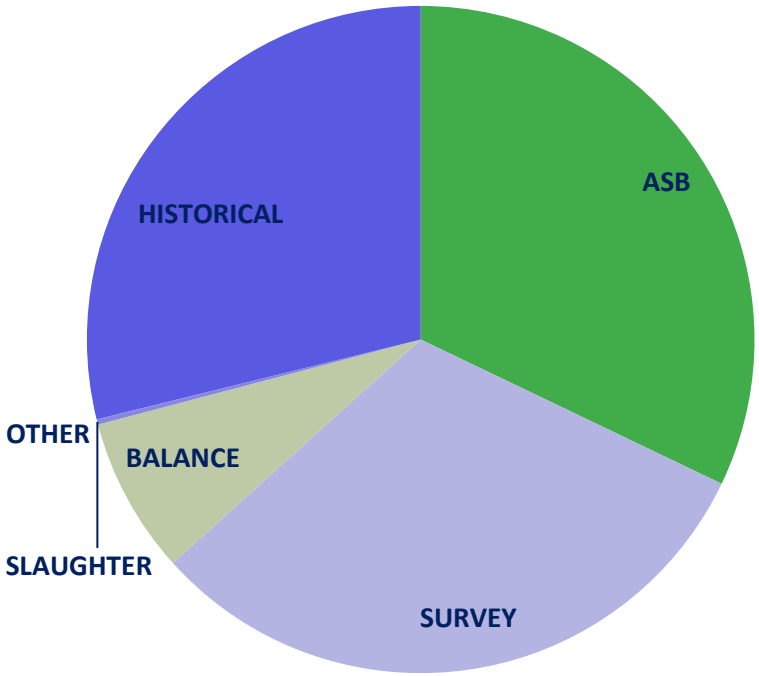


Filter Results: Total Hogs & Pigs

Contribution to Last Estimate: ASB Biased

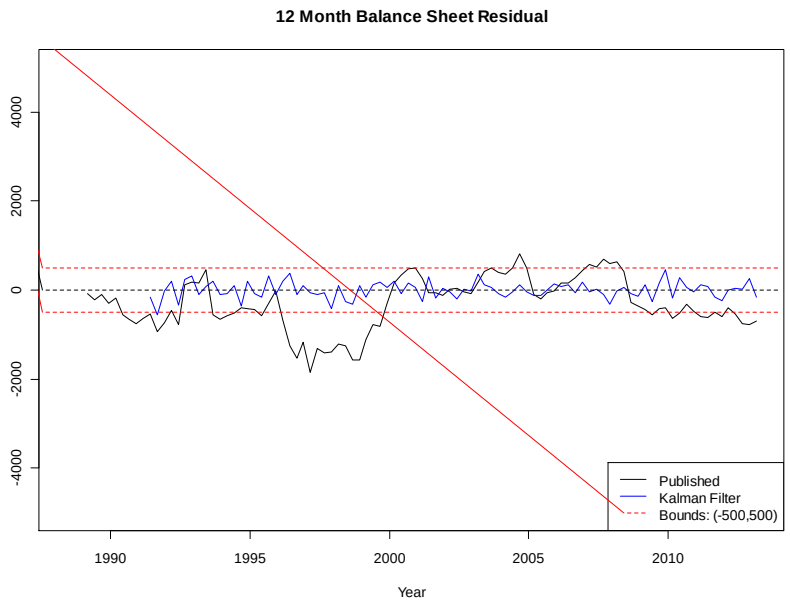


Contribution to Last Estimate: ASB Unbiased

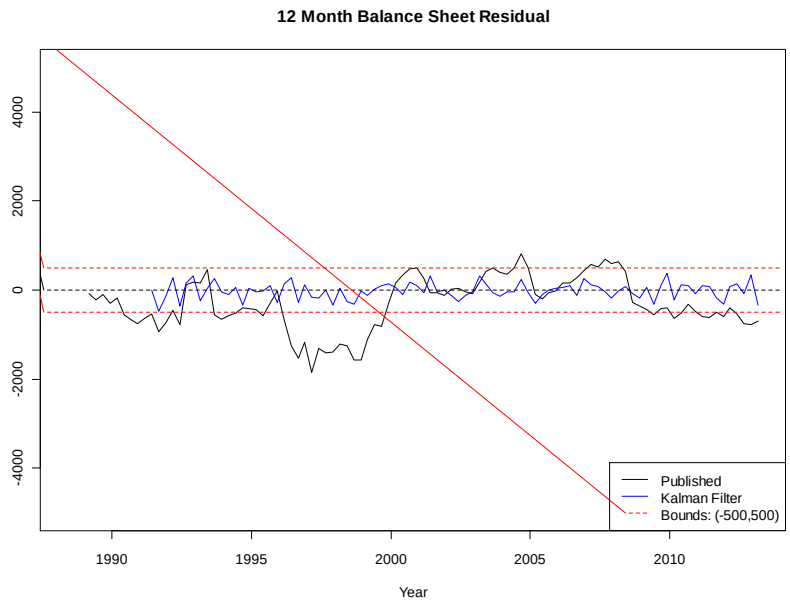


Filter Results

ASB Biased



ASB Unbiased



Additional Topics

- Handling of ASB Measurements
  - Include
    - Biased
    - Unbiased
  - Exclude
- Restricted Least Squares State Allocation
- Comparison of Kalman Filter estimates under various parameterizations

References

Box, G.E.P., Jenkins G.M., and Reinsel G.C. (2008). *Time Series Analysis: Forecasting and Control.* Hoboken, N.J.: John Wiley & Sons, Inc.

Rencher, Alvin C. (2002). *Methods of Multivariate Analysis*. Danvers, MA: Wiley-Interscience.

Shumway, R.H. and Stoffer, D.S. (2006). *Time Series Analysis and its Applications.* New York: Springer Science+Business Media, LLC.