Inference of Domain Parameters Through an Automatic Adjustment of Degrees of Freedom

Sixia Chen and Tom Krenzke, Westat
November 5, 2013
FCSM Conference
Washington D.C.
Outline

- Motivation
- Rules of thumb
- Simulation
- Conclusion
Motivation

- Provide a better inference for parameters associated with small domains, such as average income for blacks, with multi-stage samples
- Some software does not adjust degrees of freedom ($df$) for small domains (Lewis, 2013)
- Real-time on-line analytic systems (OAS)
  - Generate tables from microdata
  - Users have no access to microdata to help determine $df$ and apply a rule of thumb
  - Automatically adjust $df$ for submitted queries
Motivation (2)

- **Example**
  - With 100 replicates there are 99 \( df \) with the traditional jackknife variance estimator.
  - Suppose the sample includes only 20 cases in a domain, but \( df \) is not changed.
  - In an OAS, the inference would be based on 99 \( df \) for the domain because without access to the number of replicates and strata for the domain, the user could not adjust the \( df \).
Rules of Thumb

Adjusted degrees of freedom for a domain D

- Rule of thumb (RT)
  - Traditional RT for stratified designs
    - Number of primary sampling units (PSUs) (n) – number of strata (H)
  - Adjusted RT
    - Number of PSUs (nD) – number of strata (HD)
      - Taylor series linearization, traditional Stratified Jackknife (JKn)
Rules of Thumb (2)

- Traditional Stratified Jackknife (Shao and Tu (1996) and Wolter (2007))
  - Two or more PSUs per stratum (JKn)
    - Generates $m_h$ replicates per stratum
      \[
      \hat{V}_{JKn} = \sum_{h=1}^{H} \frac{m_h - 1}{m_h} \sum_{k=1}^{m_h} (\hat{\theta}_{D(k)} - \hat{\theta}_D)^2
      \]
      where \( \hat{\theta}_{D(k)} \) is the estimate after deleting the \( k \)th PSU from stratum \( h \).
    - Traditional RT = \( df \) can be approximated by \( \sum_{h=1}^{H} m_h - H \)
    - Adjusted RT1 = \( df \) can be adjusted by \( \sum_{h=1}^{H_D} m_{Dh} - H_D \)
Rules of Thumb (3)

- The Paired Jackknife (Rust and Rao, 1996) is the focus of this presentation
  - Two PSUs per stratum (JK2)
  - Generates only one replicate per stratum

\[ \hat{V}_{JK2} = \sum_{k=1}^{H} (\hat{\theta}_{D(k)} - \hat{\theta}_D)^2, \]

where \( \hat{\theta}_{D(k)} \) is the estimate after deleting the \( k \)th PSU from stratum \( h \).

- Traditional RT = \( df \) can be approximated by \( H \)
- Adjusted RT2 = \( df \) can be approximated by \( H_D \), where \( H_D \) is the number of variance strata \( H \) with at least one element in \( D \)
### Rules of Thumb (4)

**Adjusted degrees of freedom for domains – select citations**

<table>
<thead>
<tr>
<th>Select literature</th>
<th>df</th>
<th>Variance approach</th>
<th>Estimates</th>
<th>Inference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bryant (1994)</td>
<td>Satterthwaite</td>
<td>General replication</td>
<td>df</td>
<td>point est, standard error (SE)</td>
</tr>
<tr>
<td>Korn and Graubard (1999)</td>
<td>Adj RT1</td>
<td>Taylor</td>
<td>mean</td>
<td>Mean, SE</td>
</tr>
<tr>
<td>Burns et al. (2003)</td>
<td>Adj RT1</td>
<td>Taylor</td>
<td>mean, df</td>
<td>point est, hypothesis testing</td>
</tr>
<tr>
<td>Valliant and Rust (2010)</td>
<td>Trad RT, Satterthwaite</td>
<td>Taylor</td>
<td>total, ratio, df</td>
<td>point est, SE, confidence interval (CI)</td>
</tr>
<tr>
<td>This paper</td>
<td>Adj RT2</td>
<td>JK2, CJK2</td>
<td>mean, reg coef, total, df</td>
<td>point est, SE, CI</td>
</tr>
</tbody>
</table>
Simulation

- Data
  - 2011 National Health Interview Survey (NHIS) public use file that has a sample size of 33,014 of sampled adults

- Sample size
  - 30,075 after deleting cases with unknown study variables. It is treated as a finite population

- Monte Carlo
  - 1,000 samples
Simulation (2)

- **Sample design – Two-stage**
  - Stratified PPS without replacement with 2 sampled PSUs per stratum
  - Persons selected by SRS within the PSUs with the same sample size in each PSU, thus producing an overall equal probability sample
  - Combined strata1-10, strata11-20,……strata291-300 to create $H = 30$ strata
  - Each new stratum has $M_h = 20$ PSUs
Simulation (3)

- Variance approaches
  - Paired Jackknife (JK2)
  - Combined Strata Paired Jackknife (CJK2)
    - Combine the adjacent 2 strata as 1 stratum. Form 2 new PSUs (a new PSU consists of 2 original PSUs, one selected at random from each stratum)

- Degrees of freedom (2 types)
  - Traditional, adjusted

- Significance levels (3 levels)
  - 0.01, 0.05, 0.10

- Study variables (2 variables)
  - Weight and Body Mass Index (BMI)
Simulation (4)

- Domains (2 domains)
  - Black (15%), Asian (6%)

- Sample sizes for JK2
  - n=200 overall
  - JK2: 30 Strata, 60 PSUs
  - CJK2: 15 Strata, 30 PSUs

- Average sample sizes per PSU when n=200

<table>
<thead>
<tr>
<th>Domain</th>
<th>JK2</th>
<th>CJK2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overall</td>
<td>3.3</td>
<td>6.6</td>
</tr>
<tr>
<td>Black</td>
<td>0.5</td>
<td>1.0</td>
</tr>
<tr>
<td>Asian</td>
<td>0.2</td>
<td>0.4</td>
</tr>
</tbody>
</table>
Simulation (5)

- Parameters of interest (4 parameters)
  - Mean, regression coefficient, total, \( df \)

- Examine the Coverage rate
  - Equal to the percentage of times that the confidence interval based in the Monte Carlo sample contained the true parameter. Ideally, it should be equal to the nominal level (e.g., 95%)
Simulation Coverage Rates – Average BMI

 JK2

 CJK2

 Significance Level

 Simulation / Target Coverage Rate Difference

 Asian Adjusted

 Asian Traditional

 Black Adjusted

 Black Traditional
Simulation Coverage Rates – Average Weight

JK2

Simulation / Target Coverage Rate Difference

Significance Level

-0.11
-0.10
-0.09
-0.08
-0.07
-0.06
-0.05
-0.04
-0.03
-0.02
-0.01
0.00

CJK2

Simulation / Target Coverage Rate Difference

Significance Level

-0.11
-0.10
-0.09
-0.08
-0.07
-0.06
-0.05
-0.04
-0.03
-0.02
-0.01
0.00

Legend:
- Asian Adjusted
- Asian Traditional
- Black Adjusted
- Black Traditional
Simulation Coverage Rates – Regression Coefficient

JK2

CJK2

Simulation / Target Coverage Rate Difference

Significance Level

Asian Adjusted
Asian Traditional
Black Adjusted
Black Traditional
Simulation Coverage Rates – Total with BMI > 30

JK2

Simulation / Target Coverage Rate Difference

Significance Level

-0.05
-0.10
-0.15
-0.20
-0.25
-0.30
-0.35
0.00

CJK2

Simulation / Target Coverage Rate Difference

Significance Level

-0.05
-0.10
-0.15
-0.20
-0.25
-0.30
-0.35
0.00

Legend:

- Asian Adjusted
- Asian Traditional
- Black Adjusted
- Black Traditional
Simulation Coverage Rates – Totals for Asians with BMI > 30 and BMI > 25

![Graphs showing simulation coverage rates for different BMI categories and significance levels.]

- **JK2**
  - Simulation / Target Coverage Rate Difference vs. Significance Level
  - Different lines represent Asian Adjusted BMI > 25 and > 30, as well as Asian Traditional BMI > 25 and > 30.

- **CJK2**
  - Simulation / Target Coverage Rate Difference vs. Significance Level
  - Similar lines and categories as JK2.
Sample totals for BMI > 30 and BMI > 25

- For proportions, the denominator sample size (n=200) is the key
- For totals, the expected sample size in each domain is the key

<table>
<thead>
<tr>
<th></th>
<th>Black</th>
<th>Asian</th>
<th>Overall</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>30</td>
<td>13</td>
<td>200</td>
</tr>
<tr>
<td>BMI &gt; 25</td>
<td>22</td>
<td>5</td>
<td>126</td>
</tr>
<tr>
<td>BMI &gt; 30</td>
<td>11</td>
<td>1</td>
<td>55</td>
</tr>
</tbody>
</table>
Limitations of Simulation

- **Design**
  - Only considered stratified multi-stage PPS with two PSUs per strata for $df$ adjustment approach

- **Dataset** -- NHIS

- **Variables used**

- **Parameters of interest**

- **Sample selection for simulation spread evenly across PSUs within strata**
Conclusions

- Evaluated a rule of thumb for inference of domain parameters through an automated adjustment of degrees of freedom for the Paired Jackknife (JK2)
  - Under the simulation setup, there was some improvement seen in the adjusted rule of thumb $df$ over traditional rule of thumb $df$ for means, regression coefficients, and totals for larger domain
    - For a total for a very small domain (< 1%), both the traditional and adjusted did not perform well

- The benefits of the adjustment are greatest when there are few PSUs, e.g. when producing estimates for geographical domains
References

Thank you for your attention!

For more information, please contact:

SixiaChen@westat.com

TomKrenzke@westat.com