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Inference of Domain Parameters Through an Automatic Adjustment of Degrees of Freedom

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November 5, 2013
FCSM Conference
Washington D.C.

Outline

- Motivation
- Rules of thumb
- Simulation
- Conclusion

Motivation

- Provide a better inference for parameters associated with small domains, such as average income for blacks, with multi-stage samples
- Some software does not adjust degrees of freedom (*df*) for small domains (Lewis, 2013)
- Real-time on-line analytic systems (OAS)
    - Generate tables from microdata
    - Users have no access to microdata to help determine *df* and apply a rule of thumb
    - Automatically adjust *df* for submitted queries

Motivation (2)

- Example
  - With100 replicates there are 99 *df* with the traditional jackknife variance estimator
  - Suppose the sample includes only 20 cases in a domain, but *df* is not changed
  - In an OAS, the inference would be based on 99 *df* for the domain because without access to the number of replicates and strata for the domain, the user could not adjust the *df*

Rules of Thumb

Adjusted degrees of freedom for a domain D

- Rule of thumb (RT)
  - Traditional RT for stratified designs
    - Number of primary sampling units (PSUs) (n) – number of strata (H)
  - Adjusted RT
    - Number of PSUs (nD) – number of strata (HD)
      - Taylor series linearization, traditional Stratified Jackknife (JKn)

Rules of Thumb (2)

- Traditional Stratified Jackknife (Shao and Tu (1996) and Wolter (2007))
  - Two or more PSUs per stratum (JKn)
    - Generates m_h replicates per stratum

$$\hat{V}_{JKn} = \sum_{h=1}^{H} \frac{m_h - 1}{m_h} \sum_{k=1}^{m_h} (\hat{\theta}_{D(k)} - \hat{\theta}_D)^2$$

where $\hat{\theta}_{D(k)}$ is the estimate after deleting the k^{th} PSU from stratum h.

  - Traditional RT = *df* can be approximated by $\sum_{h=1}^{H} m_h - H$
  - Adjusted RT1 = *df* can be adjusted by $\sum_{h=1}^{H_D} m_{Dh} - H_D$

Rules of Thumb (3)

- The Paired Jackknife (Rust and Rao, 1996) is the focus of this presentation
  - Two PSUs per stratum (JK2)
    - Generates only one replicate per stratum

$$\hat{V}_{JK2} = \sum_{k=1}^{H} (\hat{\theta}_{D(k)} - \hat{\theta}_D)^2,$$

where $\hat{\theta}_{D(k)}$ is the estimate after deleting the k^{th} PSU from stratum *h*.

  - Traditional RT = *df* can be approximated by *H*
  - Adjusted RT2 = *df* can be approximated by H_D, Where H_D is the number of variance strata *H* with at least one element in *D*

Rules of Thumb (4)

Adjusted degrees of freedom for domains – select citations

Select literature	df	Variance approach	Estimates	Inference
Bryant (1994)	Satterthwaite	General replication	df	point est, standard error (SE)
Korn and Graubard (1999)	Adj RT1	Taylor	mean	Mean, SE
Burns et al. (2003)	Adj RT1	Taylor	mean, df	point est, hypothesis testing
Valliant and Rust (2010)	Trad RT, Satterthwaite	Taylor	total, ratio, df	point est, SE, confidence interval (CI)
This paper	Adj RT2	JK2, CJK2	mean, reg coef, total, df	point est, SE, CI

Simulation

- Data
  - 2011 National Health Interview Survey (NHIS) public use file that has a sample size of 33,014 of sampled adults
- Sample size
  - 30,075 after deleting cases with unknown study variables. It is treated as a finite population
- Monte Carlo
  - 1,000 samples

Simulation (2)

- Sample design – Two-stage
  - Stratified PPS without replacement with 2 sampled PSUs per stratum
  - Persons selected by SRS within the PSUs with the same sample size in each PSU, thus producing an overall equal probability sample
  - Combined strata1-10, strata11-20,……strata291-300 to create $H = 30$ strata
  - Each new stratum has $M_h = 20$ PSUs

Simulation (3)

- Variance approaches
  - Paired Jackknife (JK2)
  - Combined Strata Paired Jackknife (CJK2)
    - Combine the adjacent 2 strata as 1 stratum. Form 2 new PSUs (a new PSU consists of 2 original PSUs, one selected at random from each stratum)
- Degrees of freedom (2 types)
  - Traditional, adjusted
- Significance levels (3 levels)
  - 0.01, 0.05, 0.10
- Study variables (2 variables)
  - Weight and Body Mass Index (BMI)

Simulation (4)

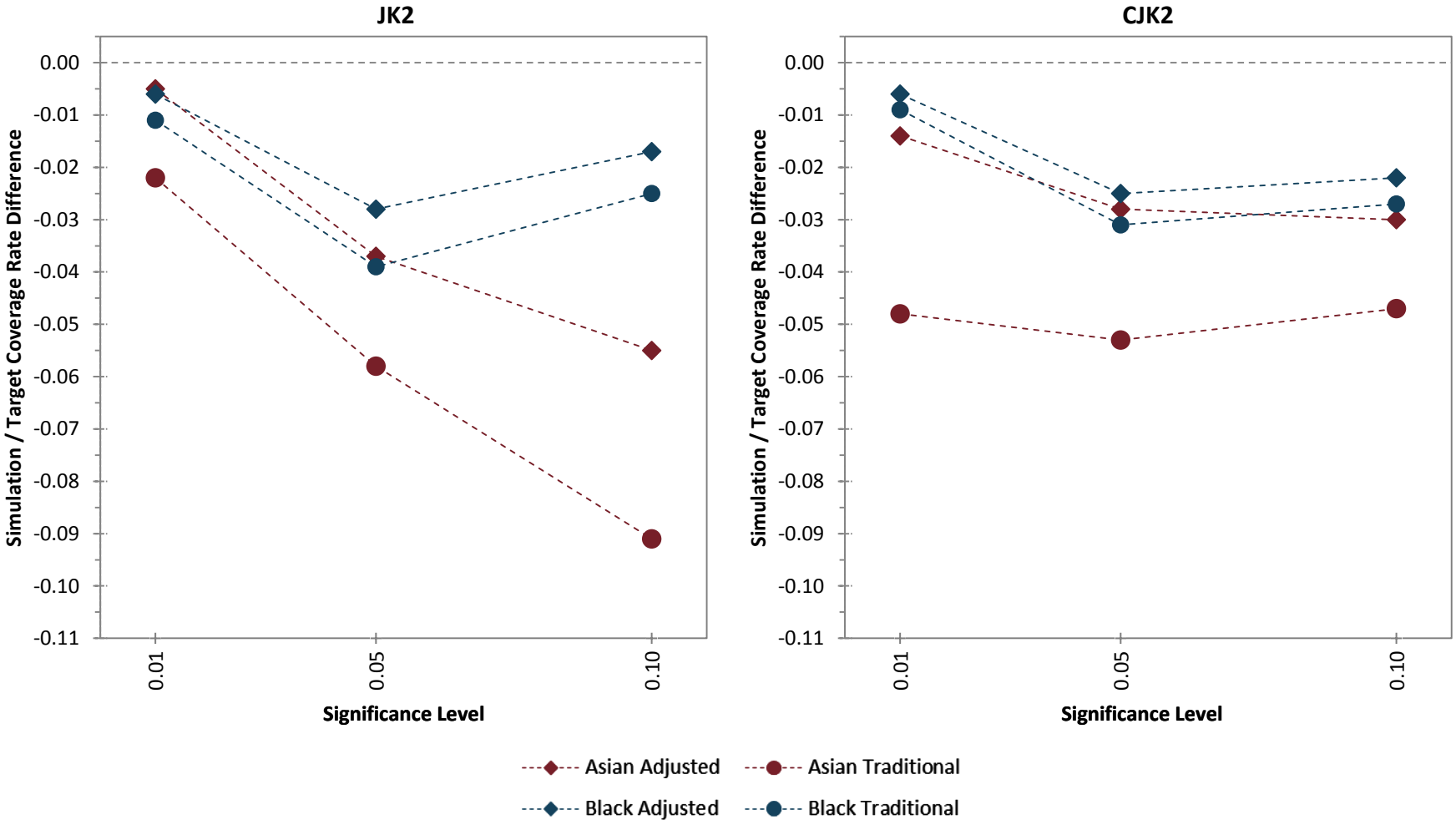
- Domains (2 domains)
  - Black (15%), Asian (6%)
- Sample sizes for JK2
  - n=200 overall
  - JK2: 30 Strata, 60 PSUs
  - CJK2: 15 Strata, 30 PSUs
- Average sample sizes per PSU when n=200

Domain	JK2	CJK2
Overall	3.3	6.6
Black	0.5	1.0
Asian	0.2	0.4

Simulation (5)

- Parameters of interest (4 parameters)
  - Mean, regression coefficient, total, *df*
- Examine the Coverage rate
    - Equal to the percentage of times that the confidence interval based in the Monte Carlo sample contained the true parameter. Ideally, it should be equal to the nominal level (e.g., 95%)

Simulation Coverage Rates – Average BMI



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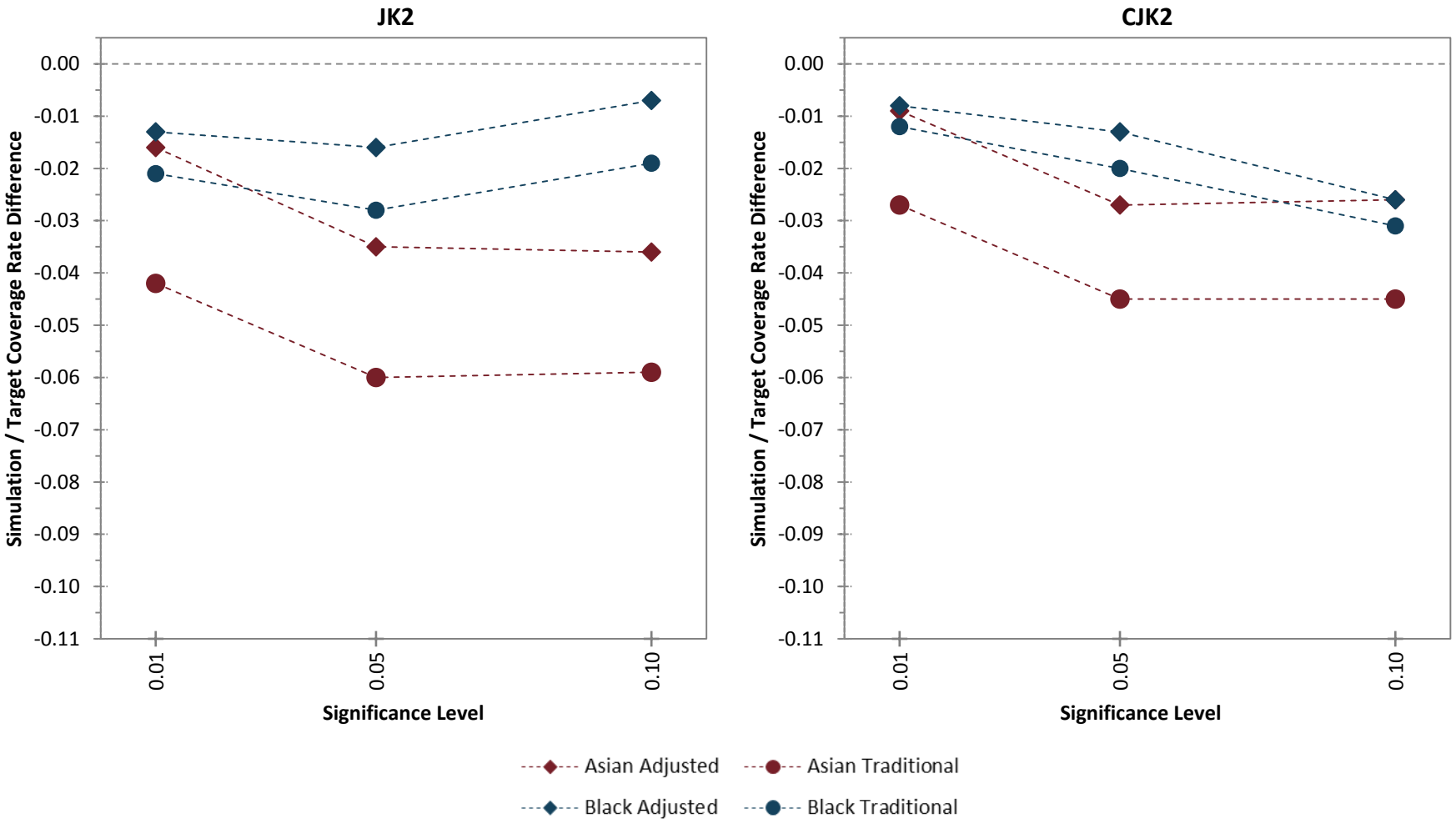
Simulation Coverage Rates – Average Weight



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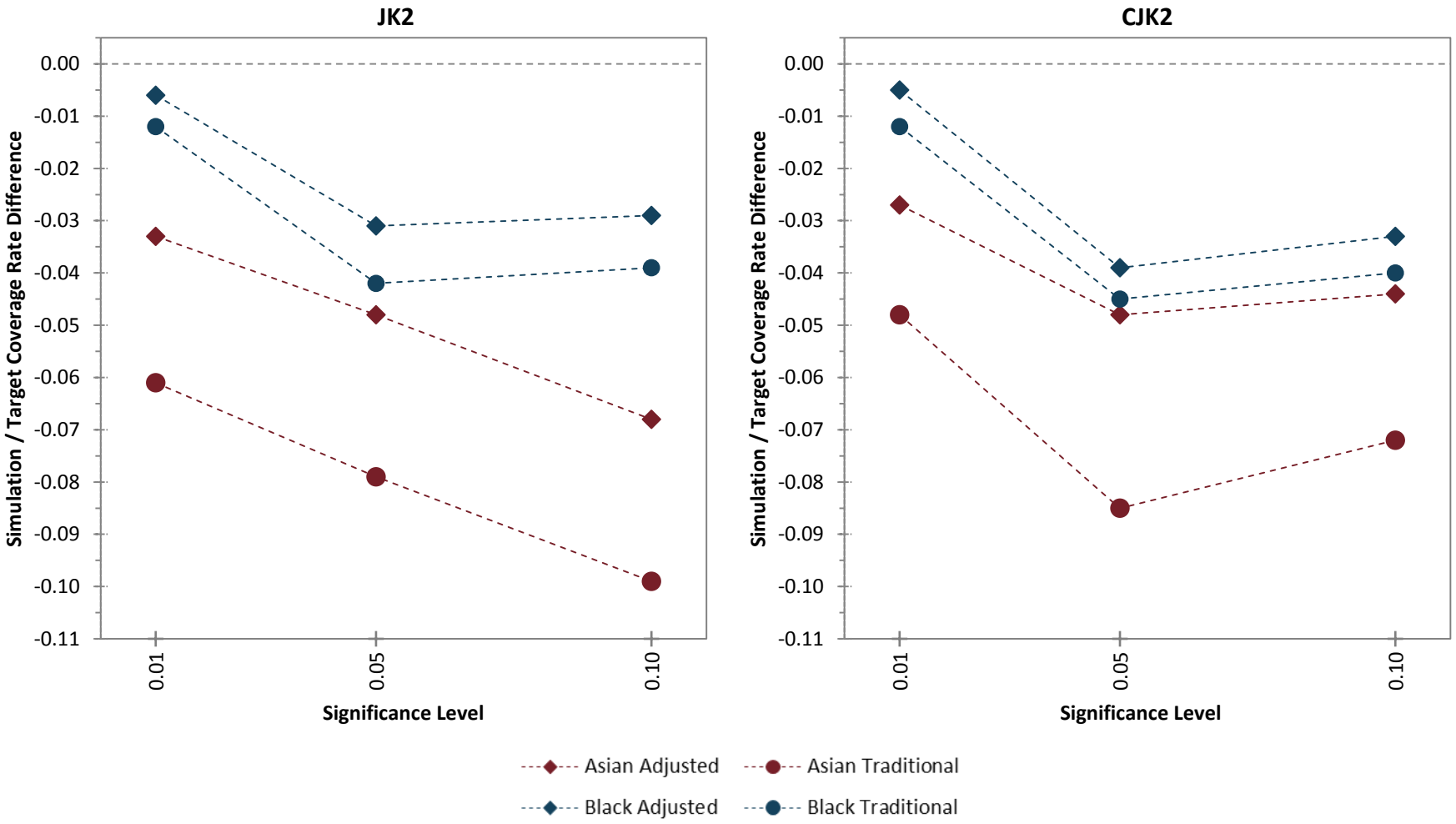
Simulation Coverage Rates – Regression Coefficient



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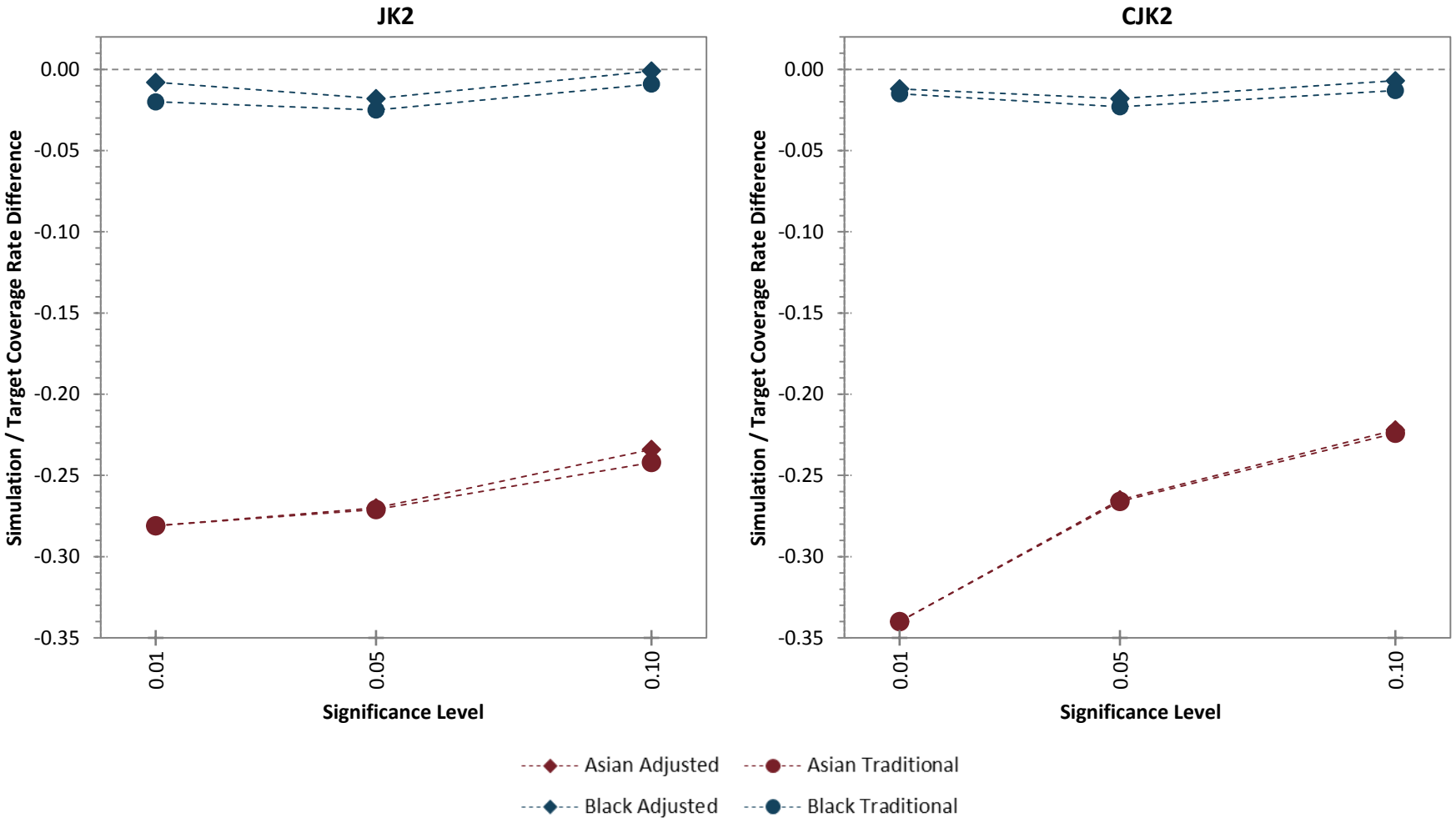
Simulation Coverage Rates – Total with BMI > 30



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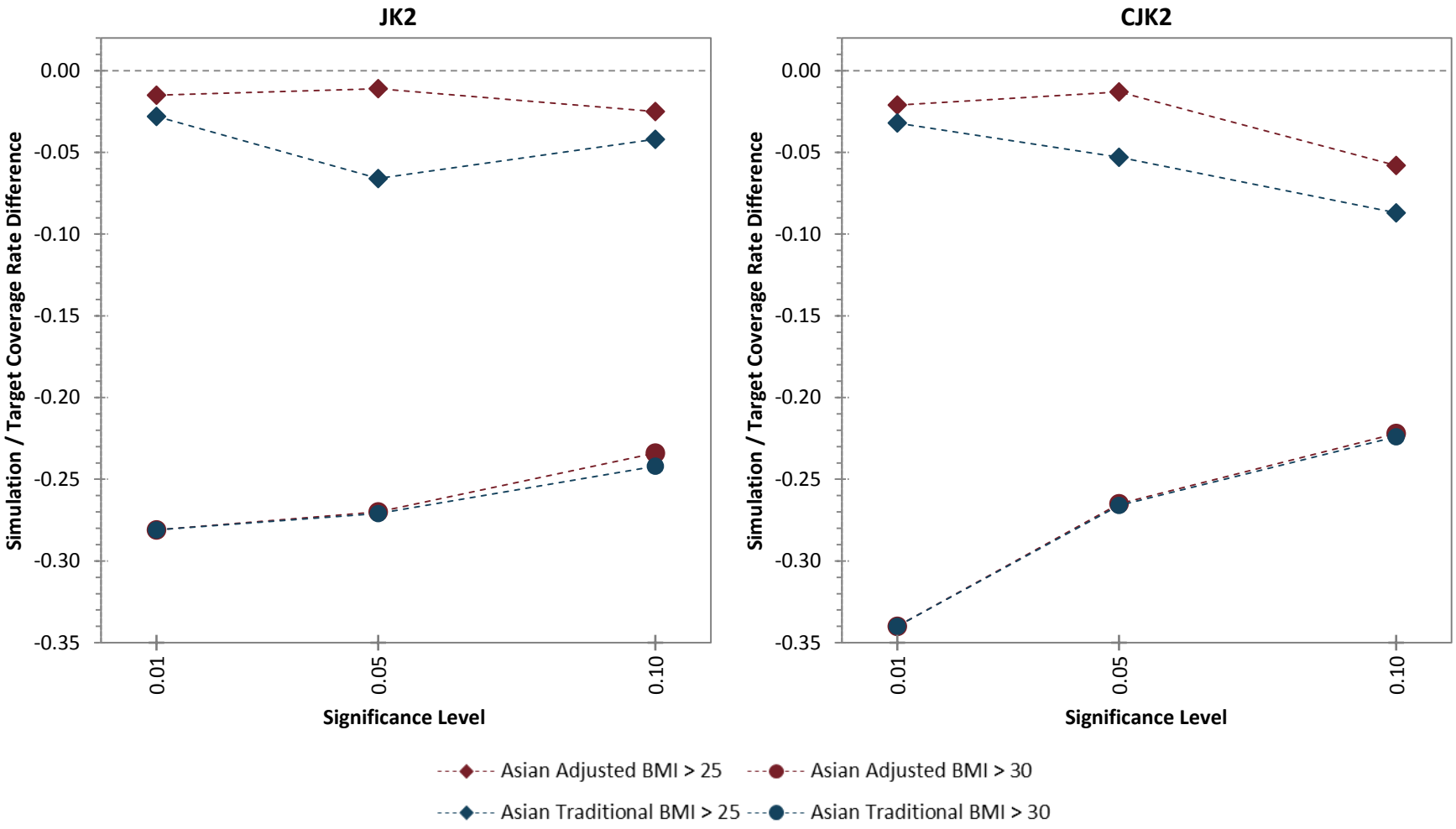
Simulation Coverage Rates – Totals for Asians with BMI > 30 and BMI > 25



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Sample totals for BMI > 30 and BMI > 25

- For proportions, the denominator sample size (n=200) is the key
- For totals, the expected sample size in each domain is the key

	Black	Asian	Overall
Total	30	13	200
BMI > 25	22	5	126
BMI > 30	11	1	55

Limitations of Simulation

- Design
  - Only considered stratified multi-stage PPS with two PSUs per strata for *df* adjustment approach
- Dataset -- NHIS
- Variables used
- Parameters of interest
- Sample selection for simulation spread evenly across PSUs within strata

Conclusions

- Evaluated a rule of thumb for inference of domain parameters through an automated adjustment of degrees of freedom for the Paired Jackknife (JK2)
  - Under the simulation setup, there was some improvement seen in the adjusted rule of thumb *df* over traditional rule of thumb *df* for means, regression coefficients, and totals for larger domain
    - For a total for a very small domain (< 1%), both the traditional and adjusted did not perform well
- The benefits of the adjustment are greatest when there are few PSUs, e.g. when producing estimates for geographical domains

References

- Bryant, E. (1994). *Methodological issues in comparative educational studies: The case of the IEA Reading Literacy Study.* Chapter 2. U.S. Department of Education, National Center for Education Statistics: Washington, D.C.
- Burns, A. M., Morris, R. J., Liu, J., and Byron, M, Z. (2003). Estimating degrees of freedom for data from complex surveys. *2003 Joint Statistical Meetings - Section on Survey Research Methods*.
- Korn, E. L., and Graubard, B. I. (1999). *Analysis of health surveys*. New York: Wiley.
- Lewis, T. (2013). Considerations and techniques for analyzing domains of complex survey data. *SAS Global Forum, Statistics and Data Analysis*.
- Rust, K. F., and Rao, J. N. K. (1996). Variance estimation for complex surveys using replication. *Statistical Methods in Medical Research,* 5, 283–310.
- Shao, J., and Tu, D. (1995). *The jackknife and bootstrap*. New York: Springer-Verlag.
- Valliant, R., and Rust, K. (2010). Degrees of freedom approximations and rules-of-thumb. *Journal of Official Statistics*, 26, 585-602.
- Wolter, K. M. (2007). *Introduction to variance estimation, 2nd ed*. New York: Springer-Verlag.

Thank you for your attention!

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