Small Area Modeling of County Estimates for Corn and Soybean Yields in the US

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Background

County Agricultural Production Survey (CAPS) Program

- In 2009-2010 the County Estimates Pilot Study was initiated in 5 States.
- New approach for selecting and processing county estimates, consistent with procedures used for the Agricultural Survey programs
- County estimates data are merged with year-end survey data.
- From 2011 onward, the probability design was expanded to cover all States.

Motivation

- CAPS results are used by field offices and HQ to establish county estimates for acreage and yield (production per acre).
  - Other administrative data are examined (Farm Service Agency, remote sensing, etc.)
  - Centralized Database for examining inputs and setting estimates
- Our GOAL is to add value to this process.
  - Efficiency: increased automation?
  - Added functionality: SE's and CV's for estimates?
- We examine *Small Area Models* as a possible solution.

Methods

- The methods described in this presentation are considered *model-based* (Rao, 2003).
  - Nested error (mixed model) regression
  - Combine survey and covariate information.
  - Induce a correlation between small areas, pooling information across the areas.
  - Referred to as "borrowing strength", a phrase ubiquitous in the literature.
- Methods are related to *shrinkage estimates*.
- Models can focus on the County (area) or Record (unit).
- Inference can be conducted with "Frequentist" and "Bayesian" approaches.

Shrinkage Estimates

- Mean Square Error ($E[Y] = \theta$)

$$\begin{aligned} E\left[(\theta - \hat{\theta})^2\right] &= \left(\theta - E[\hat{\theta}]\right)^2 + E\left[\left(E[\hat{\theta}] - \hat{\theta}\right)^2\right] \\ MSE &= Bias^2 + Variance \end{aligned}$$

- Stein's "Paradox" (Efron and Morris, 1977)
  - For $d > 2$, total MSE $\sum_{i=1}^{d} E\left[(\theta_i - \hat{\theta}_i)^2\right]$ is *not* minimzed by the Least Squares estimator $\hat{\theta}_{LS}$.
  - A class of shrinkage estimates $\hat{\theta}_{Sh}$ can be created which dominate $\hat{\theta}_{LS}$

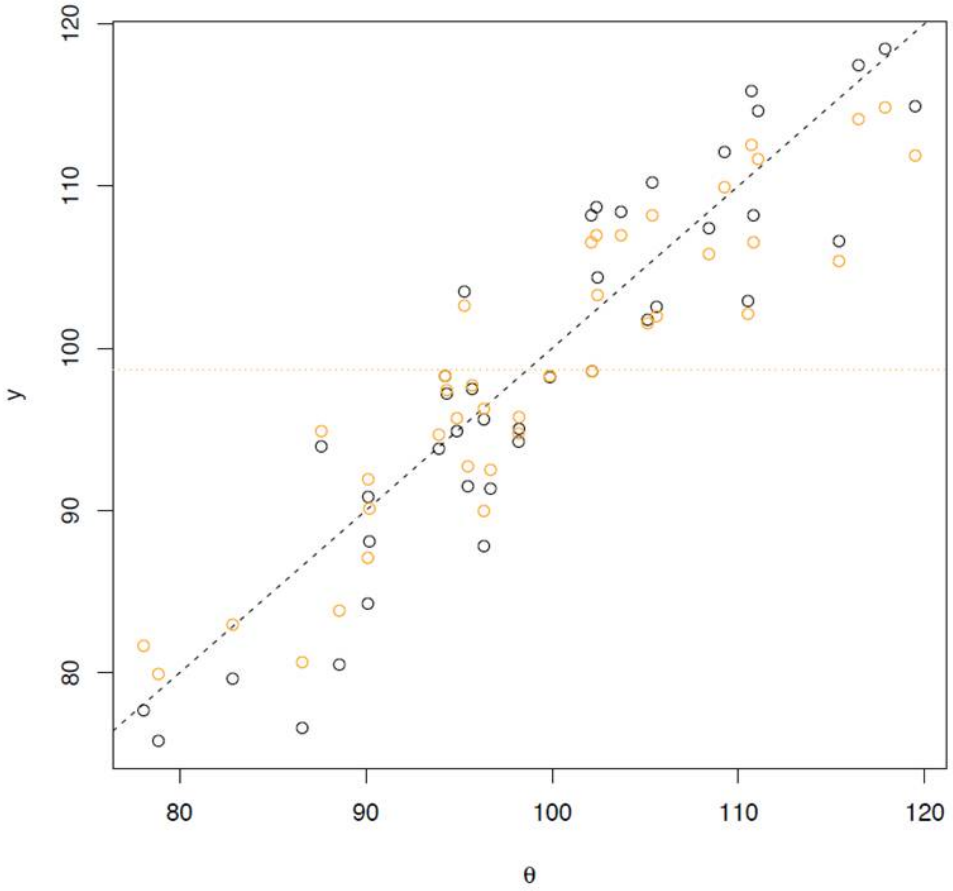
Shrinkage Estimates

- Shrinkage Estimate
  - $\hat{\theta}_{Sh} = \bar{y} + c(\hat{\theta}_{LS} - \bar{y})$
  - c can be chosen several ways.
- Composite Estimate
  - $\hat{\theta}_C = \phi\hat{\theta}_A + (1 - \phi)\hat{\theta}_B$
  - ϕ can be chosen several ways.
- Small Area Models
  - Combine two estimators: Survey Indications and Regression Model
  - Use variance components from the Survey and the Model to estimate ϕ.

Shrinkage Estimates: Example

$$\begin{aligned} \theta &\sim N(100, 10) \\ Y &\sim N(\theta, 5) \end{aligned}$$

$\hat{\theta}$	Bias	RMSE	MAD
Raw Y	33.3	29.6	155
Model	32.4	26.0	134



County vs. Record Level Models

- County (Area) Level
  - Covariates are available at the county level
  - Survey estimates and standard errors (SE's) used directly
  - Composite parameter ϕ determined by survey SE's and model estimate of between-county variance
- Record (Unit) Level
  - Covariates may be used for the record level
  - Survey estimates are modeled via record responses and weights
  - Composite parameter ϕ determined by model estimate of between-county variance and residual variance of the model (with weights)

Parameter Estimation

- Empirical Bayes
  - Frequentist approach
  - Maximum Likelihood and Restricted ML Estimation for Parameters
  - Inference uses "plug-in" estimates of parameters.
- Hierarchical Bayes
  - "Full" Bayesian approach
  - Constructs a posterior using a likelihood and a prior distribution for parameters.
  - Inference uses distributions (or samples from them) of the parameters.
  - Considerations for computation and prior selection

Results: 2010 Pilot States

- Models (14 results)
  - Area Level (EB & HB)
  - Unit Level (EB & HB)
  - Unit Level with Sampling Weights (10)
    - EB (2), HB (3)
    - With & with/out modifications (x2)

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  - National Commodity Crop Productivity Index (NCCPI)
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  - National Commodity Crop Productivity Index (NCCPI)
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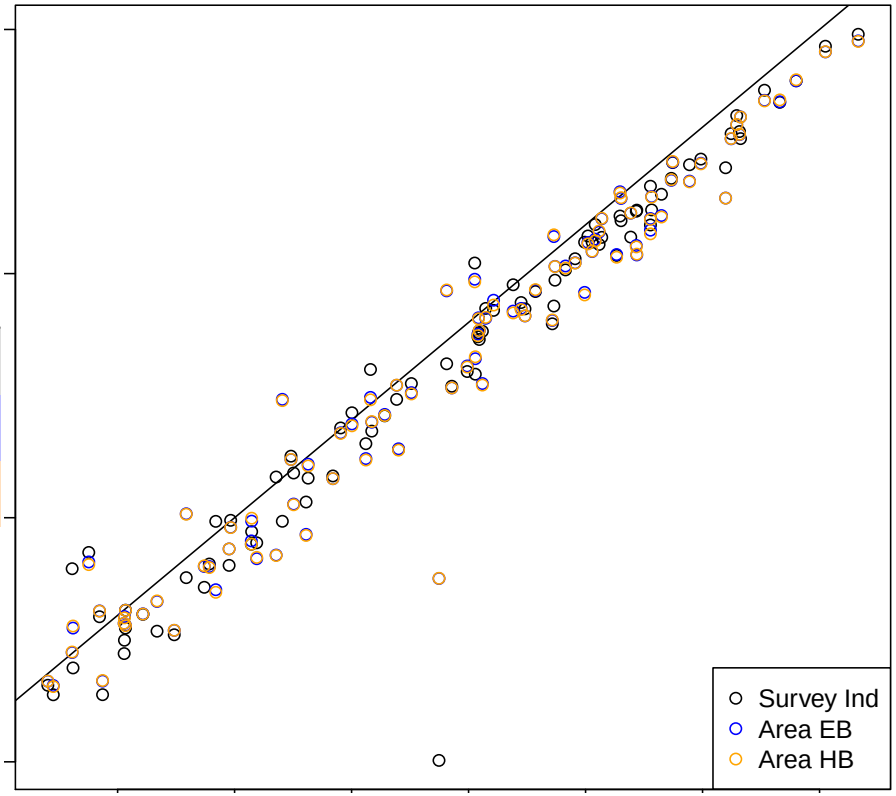
- Survey Indications
  - generally best predictor of final published values
- Area Level models
  - consistently second best
- EB and HB models
  - generally consistent
  - tighter agreement for area-level models

Results: 2011-12 Selected States

- Similar to 2010 Results
- A few states showed modest gains using Models over Survey Indications
- Some states "broke even"
- Many state Models had no improvement over the Survey Indications

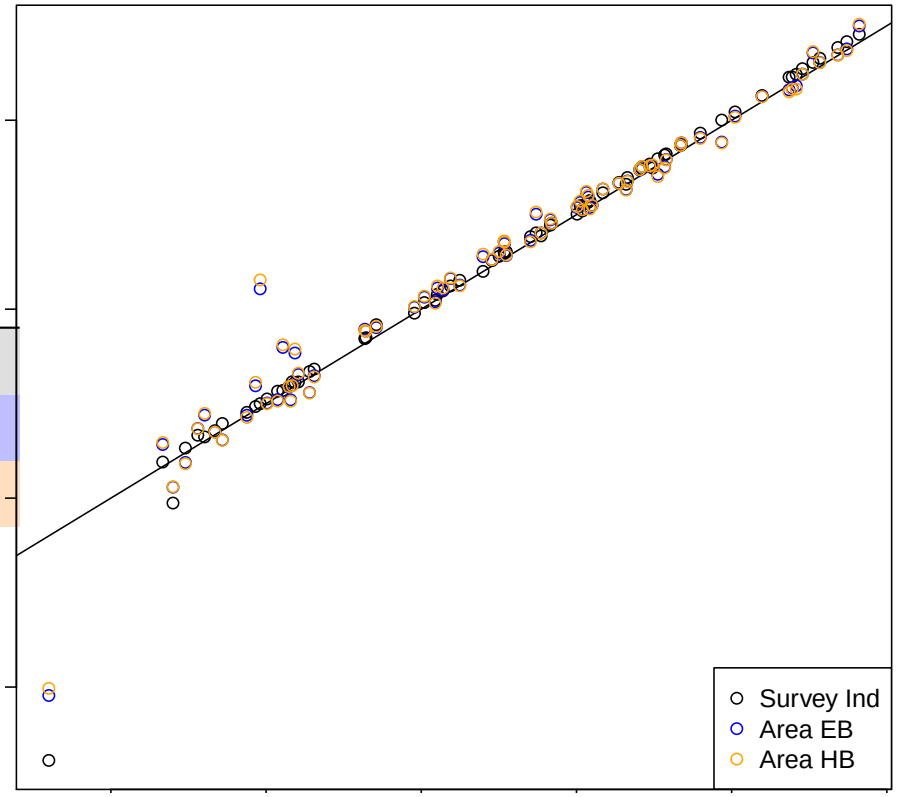
Example: Modest Gain

$\hat{\theta}$	Bias	RMSE	MAD
Direct	1.00	1.00	1.00
EB	0.94	0.81	0.99
HB	0.96	0.82	1.00



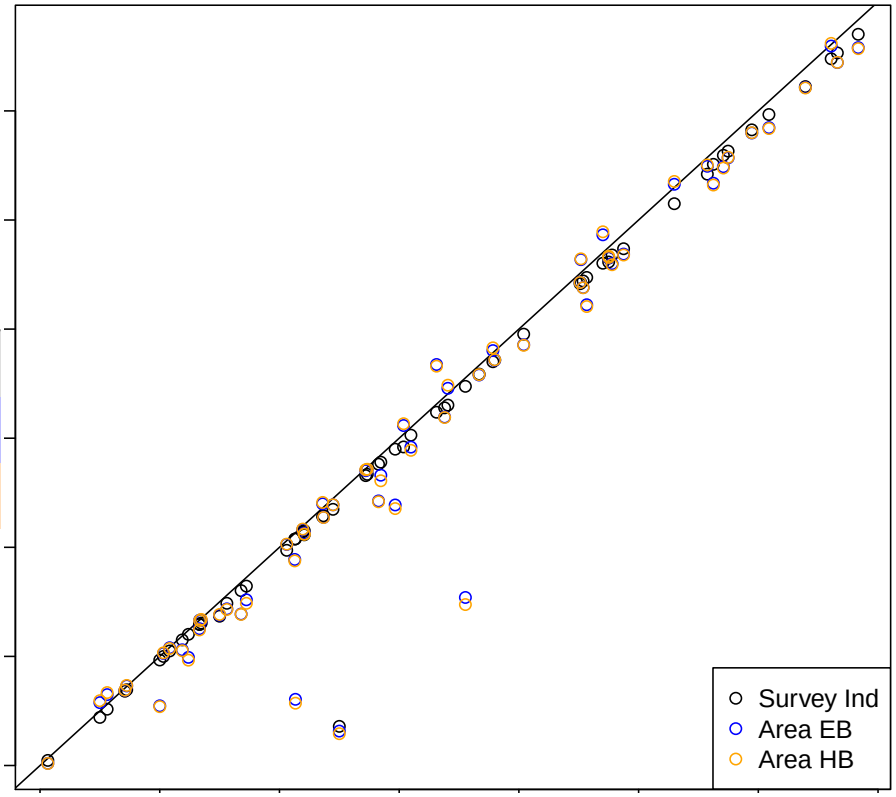
Example: Break Even

$\hat{\theta}$	Bias	RMSE	MAD
Direct	-1.00	1.00	1.00
EB	-3.25	0.93	1.47
HB	-3.38	0.96	1.56



Example: Adding Noise

$\hat{\theta}$	Bias	RMSE	MAD
Direct	1.00	1.00	1.00
EB	1.65	1.61	1.94
HB	1.68	1.65	2.02



Comments

- Final published values considered the true mean.
  - Direction of causality is reversed. Published values did not generate the survey responses.
  - Benchmarks to state values and other considerations (weather, growing conditions, etc).
  - A few large deviations (changes) and some systematic (benchmarking)
- Shrinkage estimates
  - Small amounts of change spread out over all counties
  - No decision to leave some unchanged.
- External Validation
  - How is the current county estimates program being evaluated?
  - Best available given timeliness requirement.
  - What is available later to compare? (Census 2012)

Related Research

- Benchmarking
  - County yield is a ratio. Benchmarking to state yield (ratio) is therefore nonlinear (Williams and Berg, 2013).
  - Multi-stage benchmarks and penalties (districts or variances) are also possible.
- Nonlinear relationships
  - Strong relationships between covariates and responses may be nonlinear.
  - General additive models (GAM's) have been applied to unit level models (Opsomer et al., 2008). Similar methods can be used for area level models.

References

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