Sample Size Designs to Assess Controls

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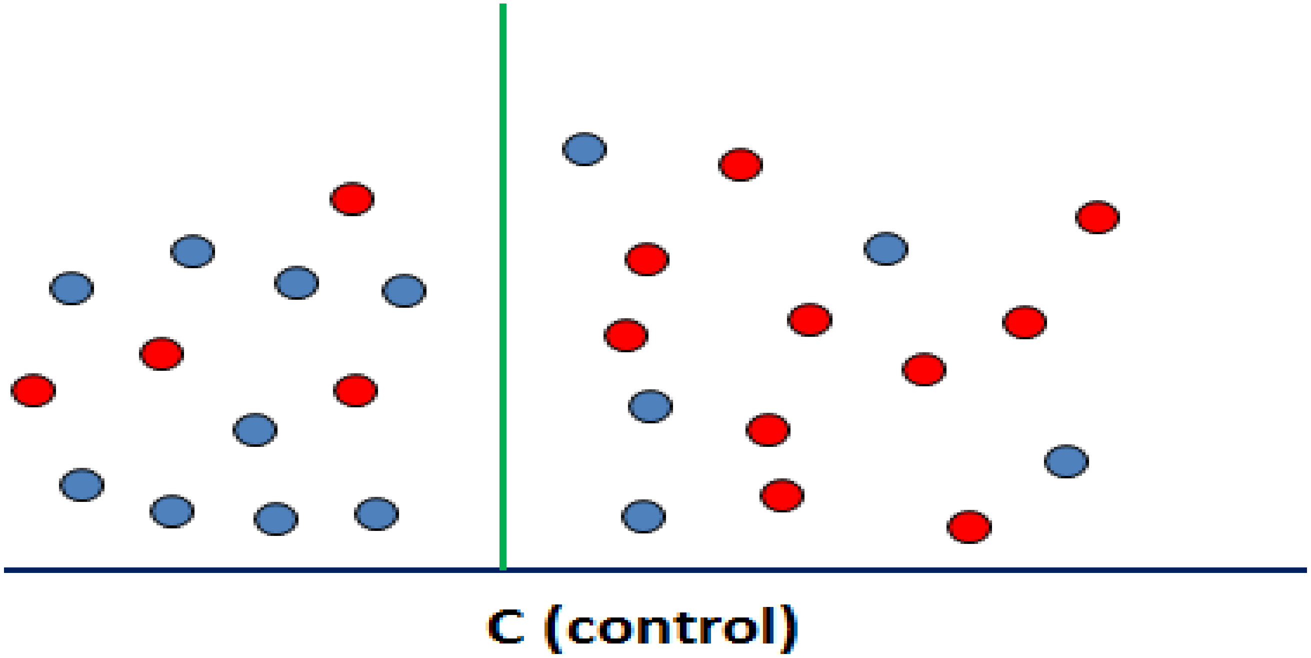
Motivation: How to Set Control?



Examples of Controls

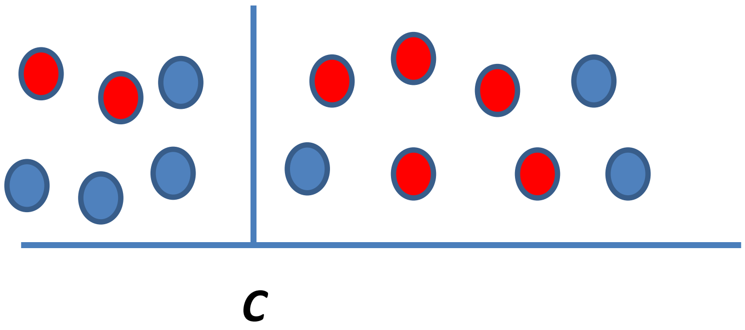
- Currency Transaction Report (CTR)
  - Flag if > US $10,000.00
- Credit score cutoffs
  - Accept / Deny applicant
- College applications and entrance exams
- Job applications and interview process
- **Challenge:** control-setting and risk

Type I vs. Type II Error Control

- Type I error: False positives
  - Compliance costs
  - Manpower
  - Resource constraints
  - Job offer to weak candidates
  - Accept weak students

- Type II error: False negatives
  - Criminal activity
  - Unfair consumer practices

Sample Design & Analysis



- *Goal:* Assess control *C*
- *Definition:* What is an "error"?
- *Q:* What sample size *n* to gauge Type II error?
- *Assumption:* "Rare error" view?
- *Result:* estimate error <u>rate</u> below *C*
- *Follow-on:* How to do deep-dives? 2-stage?

Sampling Models

- ***Hyper-Geometric*** *distribution*
  - Sampling <u>without</u> replacement model
- ***Normal*** *distribution* (approx.)
  - Cochran (1977)
  - Accommodate rare errors?
- ***Binomial*** *distribution* (approx.)
  - *n, p*
  - Any $0<p<1$; numerical solution
- ***Poisson*** *distribution* (approx.)
  - Related to Binomial

Inference I: Total Errors

- Estimation of population error rate: *p*
  - Hypothesis-testing problems & deep-dives
  - “Ballpark” notion of error incidence
  - Useful for regression modeling?
- Estimation of **<u>total</u> number of errors**: ***M***
  - Relevance to key policy decisions
  - A different view of risk
  - Informative for deep-dive studies?

Inference II: Total Errors

- Sampling models for *M* (discrete data)
  - Hyper-Geometric distribution
  - Binomial (alternative)
  - Normal (OK…could be inaccurate)

- Computations
  - Prob. of exactly ***k*** errors in a sample of size ***n***
  - Prob. of at least ***k*** errors in a sample of size ***n***

Bayesian Estimation

- **Goal:** Posterior distribution of M
  - How are total errors distributed?
  - Variability measures
  - Risk evaluations
  - Relevance for deep-dive analysis
  - Markov chain Monte Carlo

- **Alternative:** Posterior distribution of p
  - Beta-Binomial framework

Prior Distributions

- **Discrete Uniform**
  - Mass on {0,1,2,…,N}
  - Vague / Non-informative (reflects ignorance)
- **Poisson**
  - Mass on {0,1,2,3,…}
  - Leverage data
  - Empirical Bayes
- **Plan:** use *mixture* to construct prior, *p(M)*.
  - Balance “ignorance” and data.

Likelihood Function

- Quantities of interest
  - *Pr*(exactly *k* errors)
  - *Pr*(at least *k* errors)

- ***Hyper-Geometric (HG) likelihood***
  - Limited for large *N* (population size)
  - Workaround: Compute log-probabilities

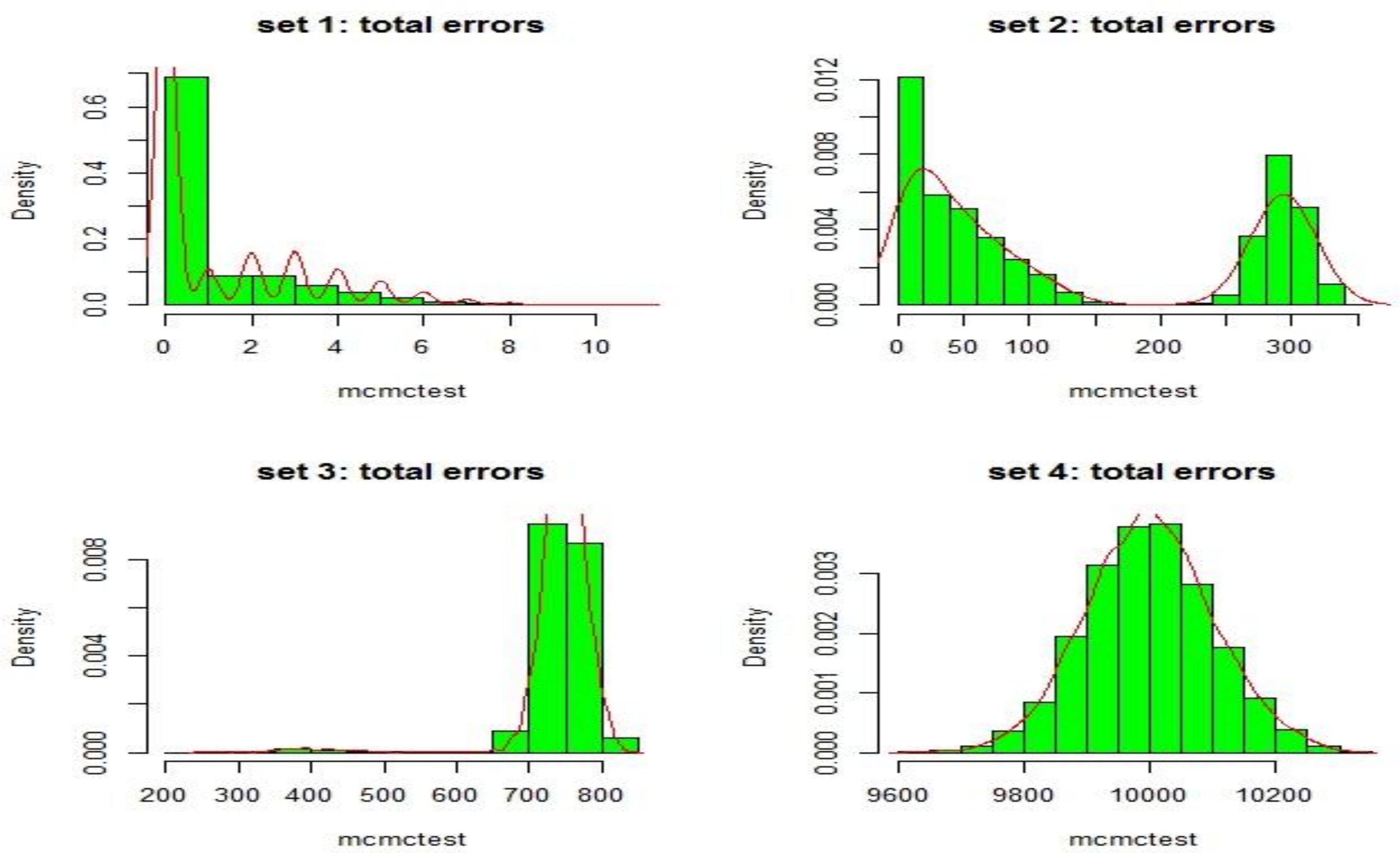
Proposal Distribution

- Poisson proposal distribution
- Choose $\boldsymbol{\lambda}$: another mixture?
- Weight $\boldsymbol{\lambda}$ (“ignorance” vs. “MCMC learning”)
- MCMC mixing properties
- Convergence of MCMC chain
- Satisfaction of ***detailed balance*** condition

Numerical Experiments

INPUTS	Set 1	Set 2	Set 3	Set 4
k (# of errors)	0	1	1	1
n (sample size)	100	100	40	100
N (pop. size)	30,000	30,000	30,000	1,000,000
Weights: Prior {DU; Pois}	{0.999; 0.001}	{0.01; 0.99}	{0.01; 0.99}	{0; 1}
Weights: Prop. {Fix; Dynamic}	{0.90; 0.10}	{0.01; 0.99}	{0.001; 0.999}	{0; 1}

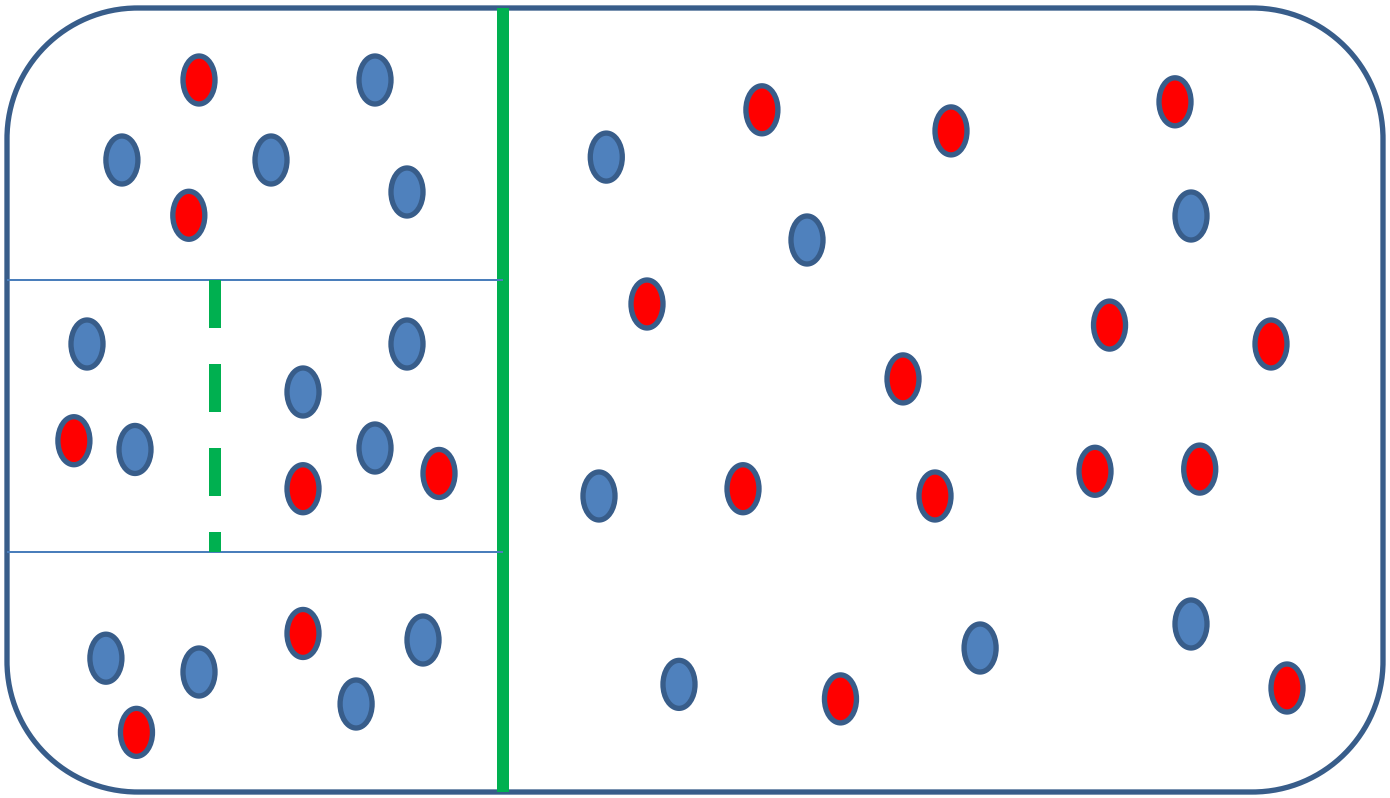
Posterior Results: Univariate Errors



Deep-Dive Sample Design

- Leverage stage 1 results for stage 2
- Posterior simulations and deep-dive
- Probabilistic view of risk
  - Easier for uni-modal posteriors
- Practical constraints vs. posterior outcome
  - E.g., bi-modal distributions
- Relevance for sample size $n = n(\text{posterior})$

Multinomial Experiment (Schematic)



Sample Size & Control-Setting

- Multinomial probabilities
  - Chi-squared tests of significance
- Deep-dive and sample size by segment
- Estimate total errors in risky sub-populations
  - Straightforward mapping to error rate
- Relevance to control settings

Multiple Levels of Errors

- Attribute sampling: binary error incidence
- E.g., degree of "injury" to consumer
- Posterior analysis
  - Bivariate HG likelihood
  - Bivariate Poisson proposal distribution
  - Co-dependence between errors?
- General modeling
  - E.g., Ordinal, Multinomial logit

Preliminary Conclusions

- Total number of errors vs. Error rate
- Posterior analysis of total errors
- Subjective / Data influence
- Sample design and posterior results
- Link control settings to risky sub-populations

Future Work

- Framework for multiple levels of errors
- MCMC estimation / posterior analysis
- Sample size and posterior results
- Logic for control settings
- Computational tool

END: THANK YOU